AstroChallenge 2020
Senior Team Round

Saturday 5th December 2020

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **16** printed pages, including this cover page.

2. Do **NOT** turn over this page until instructed to do so.

3. You have **2 hours** to attempt all questions in this paper.

4. At the end of the paper, submit this booklet together with your answer script.

5. Your answer script should clearly indicate your name, school, and team.

6. It is your responsibility to ensure that your answer script has been submitted.

7. The marks for each question are given in brackets in the right margin, like such: **[2]**.

8. The **alphabetical** parts (i) and (l) have been intentionally skipped, to avoid confusion with the Roman numeral (i).

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Question 1   Dubious Statements

This question comprises 8 statements. For each statement, indicate clearly whether it is **TRUE** or **FALSE**.

Support your answer with <u>no more than 3 sentences</u>, including any assumptions where required. You may draw up to one additional diagram if they aid your explanation.

Mathematical working is not required, and there are no errors in any of the statements below.

Each statement is worth $2\frac{1}{2}$ marks, **attributed only to the quality of the justification**.

(a) Total and annular solar eclipses cannot be observed from either of the Earth's geographic poles.   **[$2\frac{1}{2}$]**

(b) It is possible for the dust and ion tails of a comet that has entered the inner Solar System to be oriented perpendicularly in space.   **[$2\frac{1}{2}$]**

(c) Elements with atomic numbers greater than that of nickel are only formed in stars during highly energetic events. **[$2\frac{1}{2}$]**

(d) Hydrogen in some form is the dominant interstellar component in both emission and reflection nebulae, and account for the differences in their appearance.   **[$2\frac{1}{2}$]**

(e) The energy required to power the electromagnetic beams of a pulsar along its magnetic poles results in its observed period increasing continuously.   **[$2\frac{1}{2}$]**

(f) The distribution of bodies of differing masses (at different radii) in a gravitationally bound system remains unchanged with time in the absence of external forces.   **[$2\frac{1}{2}$]**

(g) Variations in the surface brightness of an observed galaxy across its cross-sectional area may be used to refine the accuracy of a distance estimate derived from a standard candle.   **[$2\frac{1}{2}$]**

(h) For an astrophotography set-up with a Newtonian telescope, removing the telescope tube in favour of an open truss design improves image contrast at the expense of image stability.   **[$2\frac{1}{2}$]**

Question 2   Brightest Day, Blackest Night

In brightest day, in blackest night,
No evil shall escape my sight.
Let those who worship evil's might
Beware my power: Green Lantern's light!

–Oath of the Green Lantern Corps
(Credit: DC Comics)

Introduction

The oath of the Green Lantern Corps is the central theme to this question. There are four parts to this question, mostly dealing with certain intricacies of the astronomically-relevant words in the oath: Brightest Day, Blackest Night, and Green Lantern's Light.

Do your best to defeat evil's (read: the question's) might!

FOR THE ENTIRETY OF THIS QUESTION, ASSUME ALL PLANETS ARE SPHERICAL AND HAVE CIRCULAR ORBITS.

Part I   Brightest Day

For this part, we only consider the Arctic circle, but with understanding that similar conclusions apply for the Antarctic circle as well. Collectively they are called the polar circles.

Polar day (or Midnight Sun) is a phenomenon whereby the centre of the Sun is visible at all times of the day. For the purposes of this question, the edge case of 'exactly on the horizon' is treated as being visible. The definition of the Arctic circle relies on this phenomenon and therefore (in theory) the circle is not fixed over time. The following presents two definitions of the Arctic circle.

> **Definition 1.** The Arctic circle is the southernmost latitude above the equator from which the centre of the Sun can be seen at all times throughout the day at least once a year, at sea level and ignoring the effect of atmosphere.

> **Definition 2.** The Arctic circle is the latitude defined by
>
> Latitude of Arctic Circle = 90° − Axial tilt of Earth.
>
> (Note: Since the axial tilt varies slightly, the value commonly quoted is typically an average over several years. For the purposes of this question, however, you should ignore this averaging.)

(a) (i)   With the aid of a **clear** diagram, justify the equivalence of both definitions, i.e. show that Definition 2 can be deduced from Definition 1, and vice-versa. Your diagram should be well-labelled.   **[2]**

(ii)   How many times does the Sun set at the (axial) poles each year? Explain why. If necessary, you may wish to draw or reuse the above answer's diagram.   **[1]**

(iii)   By definition, at the Arctic circle, polar day should occur exactly once per year on a certain day. What is this day known as?   **[1]**

In reality, however, to an observer at sea level, the phenomenon of polar day occurs up to approximately 90 km south of the Arctic circle. The main reason for this phenomenon is *atmospheric refraction*.

(b) (i)   What is atmospheric refraction?   **[½]**

(ii)   Explain briefly how atmospheric refraction gives rise to this real-world observation. You may wish to draw a suitable diagram to assist.   **[1½]**

Part II   Blackest Night

The counterpart to the polar day is the polar night, where the Sun is never visible for the entire day. The duration of polar night is defined as the total continuous duration where the Sun never rises above the horizon (for an observer at sea level). The duration of polar day is similarly defined.

(c) As mentioned above, polar day can occur outside (i.e. at latitudes closer to the equator than) the polar circles. Answer the following two questions.

   (i)   Can the polar night ever occur outside (or on) the polar circles? Explain.   **[1]**

   (ii) At a given latitude, which is more: the number of polar days, or the number of polar nights? Or are they the same? Explain.   **[1½]**

Part III   No (Part of) E.V.I.L. Shall Escape My S.I.G.H.T.

NOTE: IN THIS PART, WE ONLY CONSIDER NORTHERN LATITUDES.

A certain scientific villain wants to create a base on the airless planet E.V.I.L. (Extremely Very Irritating Lair, because which self-respecting villain doesn't have a whole planet as an evil lair that's very irritating to infiltrate?). This planet orbits a (fictional) star S.I.G.H.T. (Super Irritating Ginormous Hot Thing, because that's totally how you explain a star badly).

E.V.I.L.'s axial tilt is φ, its orbital period is T, and its rotational period is p. You may assume that $T \gg p$. Its radius is R. At time 0, the planet's axis forms an angle $\frac{\pi}{2} - \varphi$ with respect to the planet-star line, with North tilted towards S.I.G.H.T.

At time t, the angle τ between E.V.I.L.'s axis and the planet-star line is given by

$$\tau = \cos^{-1}\left(\sin\varphi \cos\frac{2\pi t}{T}\right), \qquad \frac{\pi}{2} - \varphi \leq \tau \leq \frac{\pi}{2} + \varphi.$$

Note that an angle above $\frac{\pi}{2}$ represents the Northern hemisphere tilted away from S.I.G.H.T.
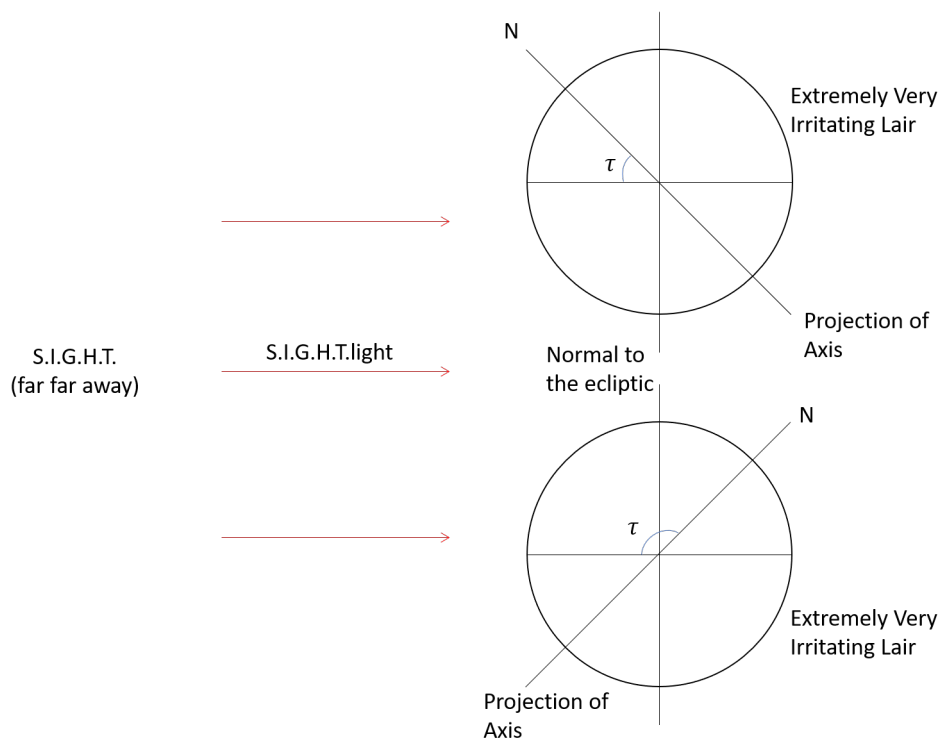

Figure 2: Cross-sectional diagram representation of τ at two different times. Note that the tilt τ of the projection of axis line changes with t. When $t = 0$, $\tau = \frac{\pi}{2} - \varphi$.

The goal of this part is to derive an expression for length of time spent in direct S.I.G.H.T.light. Or if you prefer, length of daylight.

(d) **(BONUS)** Prove the given expression for τ.   **[2]**

(e) Let L be a given (fixed) northern latitude. The points on E.V.I.L. with latitude L form a circle around E.V.I.L. This circle has radius r and is at a distance h from the equatorial plane. With a diagram, explain why we have

$$h = R \sin L \qquad \text{and} \qquad r = R \cos L.$$

   [1]

(f) Let $\mathscr{D}$ be the disk on E.V.I.L. formed by the circle in part **e**. Let ℓ be the normal line to the ecliptic that passes through the centre of E.V.I.L. Notice that depending on τ and L, ℓ may not intersect $\mathscr{D}$.

Suppose for the moment that ℓ intersects $\mathscr{D}$ at a point P.

   (i) Let d be the distance from the centre of $\mathscr{D}$ to P. Show that

$$d = R \sin L \cot \tau.$$

Again, $d < 0$ means that the Northern hemisphere is tilted away from S.I.G.H.T.   **[1]**

   (ii) Hence or otherwise show that at latitude L and position at time t, the length of time D in direct S.I.G.H.T.light is

$$D = \frac{p \cos^{-1}(-\tan L \cot \tau)}{\pi},$$

where the inverse cosine is taken over $[0, \pi]$.

(Hint: Use the fact that $\pi - \cos^{-1} x = \cos^{-1}(-x)$.)[1]   **[1]**

(g) Now we consider what happens when ℓ does not intersect $\mathscr{D}$. This only occurs for latitudes $L > \frac{\pi}{2} - \varphi$. In this part, for simplicity consider only $0 \le t \le T$.

   (i) Show that polar day occurs at a fixed latitude $L > \frac{\pi}{2} - \varphi$ when

$$0 \le t \le Q \qquad \text{or} \qquad T - Q \le t \le T,$$

where $Q = \frac{T}{2\pi} \cos^{-1}(\csc \varphi \cos L)$. Here the inverse cosine is taken over $\left[0, \frac{\pi}{2}\right]$.   **[1]**

   (ii) The same logic above (or an even simpler argument) can be used to derive a similar range of t during which polar night occurs.

Hence or otherwise, copy and complete the following expression for the length of day D at time t, in terms of p, L, τ, T, and Q:

$$D = \begin{cases} \underline{\qquad}, & \text{if } L > \frac{\pi}{2} - \varphi \text{ and } (\underline{\qquad} \text{ or } \underline{\qquad}), \\ \underline{\qquad}, & \text{if } L > \frac{\pi}{2} - \varphi \text{ and } \underline{\qquad}, \\ \underline{\qquad}, & \text{else.} \end{cases}$$

(Note: Proof for the range of t during polar night is not required.)   **[1]**

(h) This certain scientific villain needs a place that experiences lots of time in direct S.I.G.H.T.light over one orbit. At what latitude(s) should he place his base? Or is his quest to find the best location a waste of time? Justify.

(Note: Remember that E.V.I.L. is airless.)   **[1½]**

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[1]For the math buffs. From $\cos(\pi - \theta) = -\cos\theta$, by writing $\theta = \cos^{-1} x$ in $[0, \pi]$, we get $\cos(\pi - \cos^{-1} x) = -x$. Taking $\cos^{-1}$ in $[0, \pi]$ on both sides gives $\pi - \cos^{-1} x = \cos^{-1}(-x)$. Magic! C'est magnifique!

Part IV   Green Lantern's Light

Sadly, outside of special effects, we will never see Green Lantern's light (although if you do, something bad is probably happening so you should run anyway). Thankfully, the Northern lights are here to provide a beautiful alternative! Commonly known as Aurora borealis, these lights are commonly seen in winter, spring, and autumn, inside and around the Arctic circle.

The lights are typically green, although they can appear in other varieties of colour. They typically form in an irregular oval (the aurora oval) centred roughly on the North magnetic pole. They are formed when particles, typically electrons and protons, from the solar wind interacts with particles in the atmosphere.

(j) (i)   Near the Arctic circle, why are the lights not commonly seen in summer?   **[1]**

    (ii)   Explain the mechanism of formation of the Northern lights.   **[4]**

Question 3   Astronomical Project – Galaxy Morphology

Ever wondered how professional astronomers tackle questions on the universe? In this question, we will attempt to explore that by role-playing as a team of professional astronomers! Note that it is always good to tackle a problem as a team as no one is an expert at everything. Some of us are better at some skills, or are more knowledgeable than others.

For our own AC2020 line-up of astronomers (QMs), we have:

Without further ado, let us start our journey as a team of aspiring astronomers working together to unveil the mysteries of the universe!

Preamble

In view of large photometric surveys such as the Sloan Digital Sky Survey (SDSS), astronomers have been using a well-known correlation between galaxy colour and the galaxy's morphology to infer the morphological types of galaxies. The term 'morphology' is just a more sophisticated way of describing the 'type' of galaxy (i.e. spiral, elliptical, etc.).

In this project, we will be verifying the following claim:

> According to this colour-morphology relation, spiral galaxies are statistically bluer, disk-dominated, and hold more star formation than their elliptical counterparts.

Part I   Astronomical Preliminaries

First off, we will kick off the project with a quick refresher course from our local theoretical astrophysicist!

(a) The crux of this project involves investigating the colours of the galaxy.

     (i)   Briefly explain what causes the colour of different galaxies to be different.   **[1]**

     (ii)   Using your answer in part (**i**), suggest a reason why spiral galaxies tend to appear bluer as compared to their elliptical counterparts.   **[2]**

Part II   Data Collection and Image Processing

Next, we will have our amazing expert in practical astronomy, here to spearhead the collection of data! This is done by compiling galaxy images and data from various sources, such as the SDSS etc.

As suggested by our expert Grace, one quick way to classify if a galaxy is spiral or elliptical is by using the quantity $fracdeV$.

Understanding $fracdeV$

If we look at the bulges of a spiral galaxy, or at large elliptical galaxies, the surface brightness $\mu(r)$, where r represents the distance of a point located in the vicinity of the galaxy as measured from the centre of the galaxy, typically follows a $r^{\frac{1}{4}}$ law known as the de Vaucouleurs fit. This is described by the following equation:

$$\mu(r) = \mu_e + 8.3268\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right) \tag{1}$$

However, for galaxies more similar to disks, the intensity $I(r)$ of these galaxies is believed to follow an exponential decay:

$$I(r) = I_e \exp\left(-0.436\left(\frac{r}{r_e} - 1\right)\right) \tag{2}$$

Note that in equations (1) and (2), I_e, μ_e, and r_e are constants. They may or may not have physical interpretations. Also, note that $I(r_e) = I_e$.

When finding the best fit to model the light intensity profile of a galaxy, in almost all cases some combination of both curves above are required. The quantity $fracdeV$ represents the accuracy of describing the light intensity profile with only the de Vaucouleurs fit (i.e. with only equation (1)). It takes a value between 0 and 1 inclusive. A value of 1 represents a complete fit with equation (1), and 0 represents a complete misfit with equation (1) (i.e. the best fit to the light intensity profile is a pure exponential fit).[2]

(b) According to Grace, $fracdeV$ is a good indicator to use to classify a galaxy as elliptical or spiral. In her words:

> **"The smaller the value of $fracdeV$ - that is, the closer it is to 0 - the more likely it is to be a spiral galaxy."**

Using the information above, explain why this **might possibly** be true.   **[1]**

Luminosity and Brightness

For ease of comparison, we rewrite equation (1) into a form similar to equation (2), as follows:

$$I(r) = I_e \exp\left(-b\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right)\right), \tag{3}$$

where b is a constant to be determined.

The numerical value of the surface brightness μ of an extended astronomical object is defined as

$$\mu(d) = m - 2.5\log_{10} A + 5.0\log_{10} d, \tag{4}$$

where A is the total surface area of the object, m is the integrated apparent magnitude of the object, and d is the distance between the galaxy and us (Earth).

Here, integrated apparent magnitude is obtained via comparing the combined luminosity of all components of the extended astronomical object with point sources on the magnitude scale. **This means that the usual formulas for apparent magnitudes can be applied in this case**.

(c) There are two different distances, namely, r and d. Recall that

- r is the distance between the centre of the galaxy to a point located in the vicinity of the galaxy, and

- d is the distance between the galaxy and us (Earth).

A fellow astronomer is confused with the subtle differences in these two distances, and claims that an equivalent form of equation (4) holds with d replaced by r. That is, this astronomer claims that

$$\mu(r) = m - 2.5\log_{10} A + 5.0\log_{10} r.$$

Explain why this equation does not make physical sense.   **[1]**

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[2]More precisely, the light intensity profile is best approximated by a linear combination of the two equations. $fracdeV$ is the fraction of this best fit linear combination contributed to by equation (1).

(Friendly notification: The following two parts might not be for the faint-hearted!)

(d) With the help of equation (4) and relevant formulas from the Formula Book, show that equation (4) implies that

$$L = k \exp\left(-\frac{\ln 10}{2.5}\left(\mu - 5\log_{10}\frac{d}{10} + 2.5\log_{10}A - 5\log_{10}d\right)\right),$$

where all distances are measured in parsecs, L is the luminosity of the galaxy, M is the absolute magnitude of the galaxy, μ is the surface brightness of the galaxy, and k is a constant.

(Hint: Two formulas from the Formula Book are relevant. One relates absolute magnitude and apparent magnitude, and the other relates absolute magnitude and luminosity.) **[2]**

(e) **(BONUS)** Show that equation (3) follows from equation (1) under certain physical considerations. Consequently, show that $b \approx 7.67$. **[2]**

Part III   Data Analysis on Galaxy Morphology

Next, our computational expert is here to guide us through the process of analysing the data collected by Grace!

Let us recall the objective of this project:

> According to this colour-morphology relation, spiral galaxies are statistically bluer, disk-dominated, and hold more star formation than their elliptical counterparts.

The Method of Regression

To aid us in analysing this relationship, we will make use of a statistical model known as *logistic regression*[3]. Using such a statistical model, given a particular spiral galaxy's $fracdeV$, we can infer a probability that the galaxy is red (or blue). In this part of the question, we will investigate the correlation between fracdeV of a spiral galaxy with the colour of the spiral galaxy itself. Recall that in general, spiral galaxies have a small $fracdeV$ value. A larger $fracdeV$ value corresponds to one whereby the bulge of the spiral galaxy takes up a larger proportion of the galaxy itself.

A simplistic way to attempt a regression by plot is to use an indicator variable specifying if a spiral galaxy is red or blue. For instance, we could set

$$Y = \begin{cases} 1, & \text{if the galaxy is red, and} \\ 0, & \text{if the galaxy is blue.} \end{cases}$$

We can find the best fit line for Y against the $fracdeV$ of the galaxy via a simple linear model $Y = m(fracdeV) + c$. However, while we can certainly find m and c, this method does not make sense! There will be values of $fracdeV$ which are physical but Y is not. For instance, what if $Y = 0.5$? Is that galaxy red or blue? What if $Y = -1$? What if $Y = 2$?

In logistic regression, we relate the 'logarithmic probability' of a galaxy being red with the galaxy's $fracdeV$. More precisely, we model

$$\ln\frac{p}{1-p} = m(fracdeV) + c, \tag{7}$$

where p is the probability that the spiral galaxy is red.

Even so, this poses a problem. How do we assign different values of probability p for each $fracdeV$ value to determine values of m and c?

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[3]Regression refers to the process of estimating the relationship between a dependent and an independent variable. For example, plotting points on a graph and obtaining the best fit straight line is a regression technique, as it is a process which finds the line best describing the (linear) relationship between the dependent and the independent variable. Logistic regression is similar to this process, but with a curve instead of a straight line.

(f) Si Chen has an idea. For each datum, he wants to assign $p = 1$ for red spiral galaxies, and $p = 0$ for blue spiral galaxies. He will then proceed to obtain a best fit line as in equation (7) to obtain the coefficients m and c. Then for each $fracdeV$ value, the probability p such that the galaxy is red can be found from this line.

Explain why this will not work.   **[1]**

Graphing the Graph

While p is indeed determined through some method using the indicator variable Y, to surmount technical difficulties[4], magic happens and you shall assume that we have done the logistic regression for you![5] The logistic regression is done by feeding in a subsample from the citizen science project, Galaxy Zoo, with a sample size of 5433.[6] The independent variable is $fracdeV$. We have found that

$$\ln \frac{p}{1-p} = 8.2(fracdeV) - 4.9. \tag{8}$$

The following table shows a subset of the subsample that was used.

Galaxy #	Indicator Variable (Y)	$fracdeV$
1	1	0.42
2	1	0.32
3	1	0.20
4	1	0.06
5	0	0.09
6	0	0.32
7	0	0.04
8	0	0.17

Table 1: Subset of the subsample used to obtain the coefficients in our logistic regression.

(g) Using the coefficients of m and c obtained in equation (8), explain why equation (8) makes physical sense in the context of spiral galaxies.

(In other words, how do you know that these coefficients are consistent with our background knowledge in galaxy morphology?)   **[1]**

(h) On the same piece of graph paper, using p as the y-axis and $fracdeV$ as the x-axis, do the following.

(i) Draw the logistic regression curve obtained in equation (8).

(Hint: What is the best way to *draw* a curve, given its equation?)   **[3]**

(ii) Plot all the points in the subset shown in Table 1. For each datum, we set $p = 1$ if the spiral galaxy is red, and $p = 0$ if the galaxy is blue.   **[1]**

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[4]At this juncture, we will ignore this issue of assigning p to avoid other unnecessarily complicated statistical terms. The basic idea is that using a set of data points (X, Y), we are able to obtain the coefficients in the logistic regression equation (7) with the use of certain numerical techniques under certain statistical considerations.

[5]Yay!

[6]Masters, K. L. et al. (2010). "Galaxy Zoo: passive red spirals." Mon. Not. Roy. Astronom. Soc. 405, 783-799. DOI: 10.1111/j.1365-2966.2010.16503.x. arXiv: 0910.4113.

Spiral Galaxy Classifier^{TM}

With our logistic regression complete, we can predict if a spiral galaxy is red or blue given its $fracdeV$. To this end, using equation (8), Si Chen has programmed a spiral galaxy classifier, named *Spiral Galaxy Classifier*^{TM}, which only has two possible outputs: "Galaxy is Red", and "Galaxy is Blue". The classifier is programmed such that given a particular $fracdeV$ value, the corresponding probability p is determined from equation (8). It then outputs "Galaxy is Red" with probability p, and "Galaxy is Blue" with probability $1 - p$.

(j) Suppose you are provided with the following image.

Figure 3: A pair of blue spiral galaxies.

You are also told that both spiral galaxies in the image above have $fracdeV = 0.3$. What is the probability that the *Spiral Galaxy Classifier*^{TM} classifies **both** galaxies correctly?   **[2]**

Part IV   Results and Discussion

As with every project, all the team members must (finally) come together to discuss about interesting outliers and insights that result. Thus, here they are below, back for the last portion of this project!
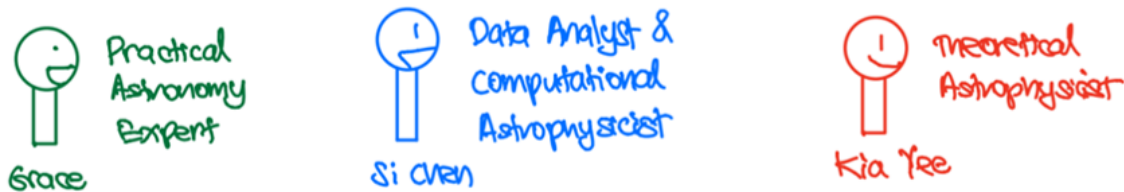
(k) From the Hubble sequence for galaxies, there is also another type of galaxy which is neither spiral nor elliptical. They are consequently classified as irregular galaxies.

Suppose that we try to classify the following galaxy using the *Spiral Galaxy Classifier*^{TM}.

Figure 4: Image of a well-known irregular galaxy, the Small Magellanic Cloud.

Is the probability of getting a "Galaxy is Red" result greater than 0.5, or is it less than 0.5? Justify your answer.   **[2]**

(m) From Table 1, we have observed a red spiral galaxy. Discuss and explain how some of these red spiral galaxies could have formed.

(Note: To score full credit for this question, you will need to give at least two possible explanations as to how these red spiral galaxies are formed.)   **[3]**

Question 4   The Comet Hunt

Comet 67P, also known as Churyumov-Gerasimenko, was first discovered in 1969 with a perihelion distance of 1.29 AU, an aphelion distance of 5.88 AU, and an orbital period of about 6.44 years.

Figure 5: Comet 67P/Churyumov-Gerasimenko. Image taken by the *Rosetta* spacecraft.

In the late 1990s, it attracted significant attention in astronomy circles when it was chosen by the European Space Agency (ESA) as one of the objects of interest in their future comet chasing mission. In March 2004, ESA launched the *Rosetta* mission, aimed at obtaining scientific data from comets such as Comet 67P.

The Ariane 5G+ rocket brought the *Rosetta* mission spacecraft to low Earth orbit (LEO). The spacecraft was then further boosted on its path towards comet 67P. It successfully reached Comet 67P in August 2014 and sent back a significant volume of important scientific data over two years.

In this question, we will explore this journey to collect scientific data from Comet 67P.

Part I   Rocketry

Ariane 5G+ is a two-stage rocket, consisting of the core stage (where the payload is contained) and the second stage. The parameters for the core stage and the second stage are given as follows:

Parameter	Core Stage	Second Stage
Length	23.8 m	3.4 m
Diameter	5.4 m	5.4 m
Empty mass (without fuel and payload)	12200 kg	1200 kg
Gross mass (fully fuelled without payload)	170500 kg	11200 kg
Engines	Vulcain1	Aestus
Thrust	1015 kN	27 kN
Burn time	605 s	1170 s
Fuel	Liquid hydrogen	Monomethylhydrazine

Table 2: Data for the two stages of Ariane 5G+.

Figure 6: A fun picture of Ariane 5G+.

(a) A rocket of mass M burns a fuel mass of ΔM in a small amount of time Δt. It then ejects the burnt fuel with velocity v_e and changes the rocket's velocity by Δv.

Draw a simple diagram illustrating this situation.

(Note: A single, well-labelled diagram with all relevant details will suffice. It is not necessary for Δt to appear in your diagram.) **[2]**

(b) By respectively using Newton's second and third laws of motion, or otherwise, state two equations relating the quantities in the diagram drawn in part **a**.

(Note: In considering the second law, you should be using the form $F = \frac{dp}{dt}$. In place of Newton's third law, you may alternatively use conservation of momentum.) **[2]**

(c) Using the thrust given and any other relevant data, calculate the exhaust velocity v_e of the fuel used for both stages. You may assume the exhaust velocity remains constant as the rocket accelerates. Ignore any atmospheric effect. You may also assume that the burn rate of the fuel is constant over the entire period of the burn.

(Note: You may find the equation(s) you have written in part **b** useful for this calculation. Remember also that $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.) **[2]**

(d) Calculate the speed of the rocket after the burning of each stage of the rocket, assuming the rocket travels vertically upwards. Take the payload weight to be the weight of the *Rosetta* mission satellite at 3000 kg.

(Note: Remember that the second stage burns first, so at the end of burning the second stage, you should be left with an empty second stage and a fully loaded core stage. The second stage then detaches and the core stage starts to burn.) **[2]**

(e) The rockets are launched due east from the Guiana Space Centre, located in French Guiana, South America, with a latitude of $5.1673°$ N, rather than in Europe where most of the parts are manufactured. Suggest a(n) astronomical/physical reason for this choice of launch location.

(Note: Practical reasons such as cheaper labour cost and better weather are **NOT** accepted.) **[1]**

(f) Contrary to some spacecraft operating entirely in space (e.g. Apollo programme's Lunar Module, Soviet Luna program), rockets (like Ariane 5G+) that take off from Earth are multistage rockets.

    (i) Suggest a reason why a multistage design is used for rockets taking off from Earth, rather than a single stage design. Support your answer with relevant calculations. **[2]**

    (ii) **(BONUS)** Suggest another reason why this is so. Calculations are NOT required in this part. **[1]**

Part II   Orbital Mechanics

Up to 1840, Comet 67P was in orbit around the Sun with a perihelion distance of 4 AU. A close encounter with Jupiter altered its orbit to one with a perihelion distance of 3 AU.

Over the next century, further encounters reduced its perihelion distance to 2.77 AU. Finally, in 1959, another close encounter with Jupiter changed its orbit to the one it currently has today, with a perihelion distance of 1.24 AU.

This proximity to Earth's orbit makes it an excellent target for scientific studies. The table below contains astrometric data of Comet 67P, obtained in 2014 as part of the *Rosetta* mission.

Nucleus overall dimensions	4.34 km ×2.60 km ×2.12 km
Small lobe dimensions	2.50 km ×2.14 km ×1.64 km
Large lobe dimensions	4.10 km ×3.52 km ×1.63 km
Mass	1.0×10^{13} kg
Density	533 kg/m^3
Orbital period	6.45 years
Perihelion distance from Sun	186 million km (1.243 AU)
Aphelion distance from Sun	849.7 million km (5.68 AU)
Orbital eccentricity	0.640
Orbital inclination	7.04°
Surface temperature	−93°C to 53°C

Table 3: Astrometric data of Comet 67P/Churyumov-Gerasimenko.

You are also given that its current albedo is 0.06, and that Comet 67P is likely to be a result of a fusion of two smaller comets.

(g) As mentioned above, in 1959 the orbit of Comet 67P was altered due to its close encounter with Jupiter. With a diagram, suggest a reason for and briefly explain this alteration. **[2]**

(h) The flight path of the *Rosetta* mission spacecraft was controlled by its onboard computer, which is notably much less powerful than those at the ESA Mission Control. Suggest why the onboard computer is used, and **not** the computers at the ESA. **[1]**

(j) For this question, assume that Comet 67P is a uniform sphere. The Hill radius r_H of Comet 67P denotes the distance from Comet 67P that a satellite, such as the *Rosetta* mission spacecraft, would transit from orbiting the Sun to orbiting the comet. The Hill radius is given by the formula

$$r_H = r \sqrt[3]{\frac{m}{3M}},$$

where r is the distance between Comet 67P and the Sun, m is the mass of Comet 67P, and M is the mass of the Sun.

Compute the average Hill radius of Comet 67P. **[2]**

(k) In reality, the *Rosetta* mission spacecraft does *not* orbit around Comet 67P at the distance calculated in part j. What is the **most** important reason why? **[1]**

(m) Suggest a possible reason why comets such as Comet 67P are of interest to astronomers. **[2]**

Read the following excerpt from NASA regarding air quality classifications.

'*Air quality classifications, as defined by FED-STD-209E, are specified by the maximum allowable number of particles per cubic foot (or cubic meter) of air. The name of the class in English units (the usual convention in the U.S.) is taken from the maximum allowable number of particles, 0.5 μm and larger, per cubic foot. Class 350,000 air is typically referred to as a "good housekeeping area" and is suitable for most integration and assembly operations. Class 00,000 – Class 1,000 air is referred to as a "cleanroom" and is required for installation of most space system hardware.*'

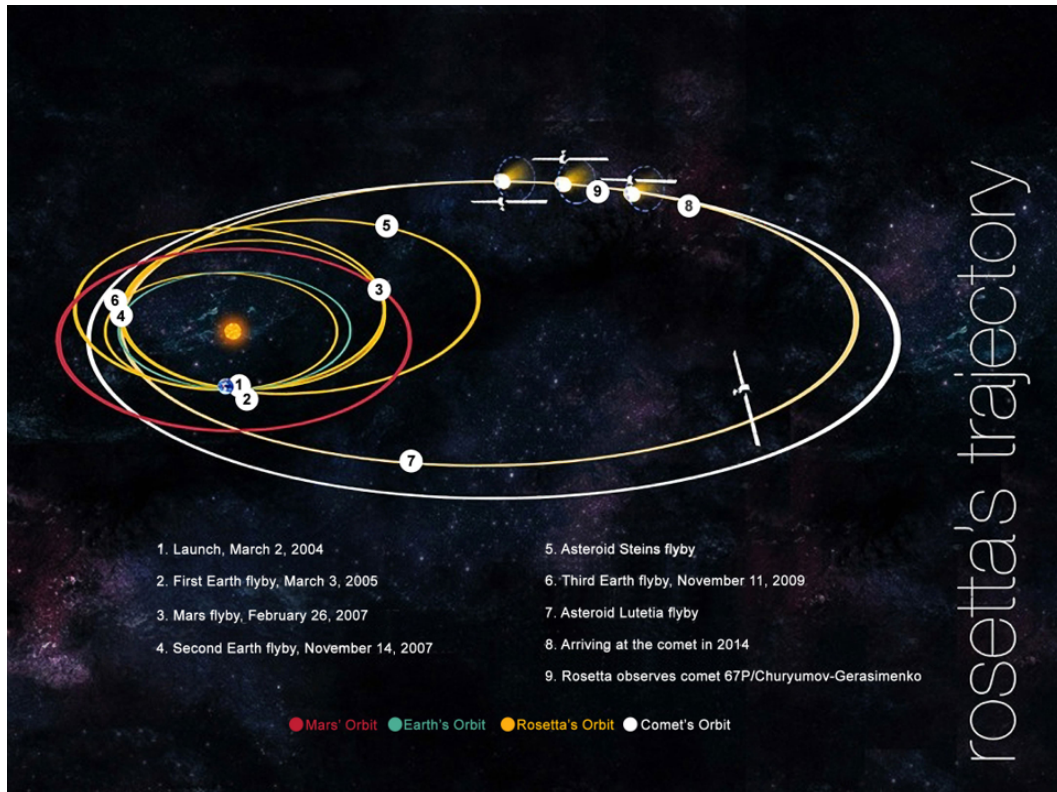Next, consider the flight path of the *Rosetta* mission spacecraft.

Figure 7: *Rosetta*'s mission flight path.

(n) Most spacecraft parts are manufactured under cleanroom conditions. The parts for the *Rosetta* mission are no exception. By referring to the flight path of the *Rosetta* spacecraft to Comet 67P in Figure 7, explain why this is necessary in the particular case of the *Rosetta* mission spacecraft. **[1]**