AstroChallenge 2020
Junior Team Round

SOLUTIONS

Saturday 5th December 2020

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **32** printed pages, including this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have **2 hours** to attempt all questions in this paper.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your name, school, and team.
6. It is your responsibility to ensure that your answer script has been submitted.
7. The marks for each question are given in brackets in the right margin, like such: **[2]**.
8. The **alphabetical** parts (i) and (l) have been intentionally skipped, to avoid confusion with the Roman numeral (i).

Question 1 A long night at Pulau Ubin

The night sky is Singapore is not a particularly amazing place for stargazing enthusiasts, yet many of the brightest stars in the night sky can still be visible. You and your friends (who do not know much about astronomy, as will be evident) are out and about in Pulau Ubin, ready to immerse yourselves in the wonders of the night sky and the buzz of mosquitoes.

(a) One of your friends points up at the sky and exclaims, "That is a very bright star!" You know better, however, and you explain her mistake and point out that that particular point of light is in fact, Venus. Of course, Venus is a planet and not a star.

How do you tell apart planets from stars (without using a star chart application)? Explain how your method distinguishes between the two. **[2]**

Solution:

Any of the following answers are acceptable:

- Stars twinkle but planets rarely twinkle. Stars appear to us a point sources of light while planets are close enough to be resolved as disks, meaning air turbulence will not disrupt their appearance as much.
- Constellations of fixed stars do not suddenly "have a new star" suddenly appear (other than novae). Being familiar with the night sky means you know when a bright point of light belongs to a constellation (and is therefore a star) or not. If it is not moving rapidly across the night sky, chances are it is a planet.

(b) Complete the following.

(i) Having corrected her, you go on to show your friends that not all stars are white points of light. Some, like Arcturus and Betelgeuse, are very visibly red. You also show them other stars that appear blue, yellow, and orange, and explain to them that this is a result of the surface temperature. Hotter stars are bluish and cooler stars are reddish, and their colour spans the rainbow.

Someone then asks you a curious question. Why are there no green stars? Your explanation is... **[2]**

Solution:

...tars with their peak intensity at green light also emit a lot of red and blue light. The human eye interprets this as white.

(ii) You manage to surprise your friends with the fact that the Sun is actually white in colour. It only appears yellow to us because... **[1]**

Solution:

...of Rayleigh scattering.

(c) Another friend, using your planet discerning method, finds that all three planets currently present in the night sky, as well as the moon, happen to lie on the same great circle. Why is this so, and what is the underlying physical reason for this?

(Note: A *great circle* is a largest circle that can be drawn on a sphere. An example is the meridian on the celestial sphere, or the equator on Earth.) **[3]**

Solution:

All the planets orbit the Sun in roughly the same plane, known as the ecliptic (or the invariable plane, approximately the same thing). This is because during the formation of the solar system, all the planets formed from the circumstellar disk, which lies in one plane. This in turn happened due to the conservation of angular momentum and the collision of particles during the gravitational collapse of the formless molecular cloud that the Sun formed in.

(d) Another friend then asks if there could be a lunar eclipse tonight, because she loves it when the Moon turns red. After disappointing her (your answer was "no"), you explain why the Moon turns red during a lunar eclipse. The reason is because... **[1]**

Solution:

Light from the Sun passes through Earth's atmosphere on the way to the Moon and shorter wavelengths of light are scattered through Rayleigh scattering, leaving only the longer, redder light to reach the Moon.

(e) (i) This same Moon-loving friend then notes that she only ever sees one side of the Moon. Tell her the name of this phenomenon and the cause of it. **[3]**

Solution:

Tidal locking/geosynchronous orbit. Earth's gravity stretches and squeezes the Moon causing tidal bulges and pulls on different parts of the Moon with different strengths. The net effect it to have the tidal bulges of the Moon be aligned to the Earth-Moon line throughout the course of the Moon's orbit, causing tidal locking.

(ii) Assuming that the Moon loses rotational momentum during the phenomenon in part (i), where or what other forms of energy has it been lost or converted to? List **three** possible avenues of energy loss or conversion. **[3]**

Solution:

Any of the following answer is accepted:

- The Moon gains orbital angular momentum
- The Earth's rotational velocity increases
- Dissipated as tidal heat/geological processes on the Moon

(f) (i) Yet another friend recognises Betelgeuse, since it was reported on the news, supposedly being close to going supernova. He worries that the Sun might do the same, as he has not fulfilled his wish of attending next year's AstroChallenge.

You comfort him by telling him that the Sun will never go supernova. He asks you why. How do you answer? **[1]**

Solution:

The Sun is too light; the Sun is not massive enough.

(ii) You then show the group the star Alpha Centauri (Rigil Kent), one of the closest stars to the Sun. Yet *another* friend then asks you how astronomers know these distances. Give **two** methods astronomers use to measure distances to visible stars in the night sky. **[2]**

Solution:

Only the follwoing 2 answers are accepted

- Parallax (any kind of parallax)
- Variable stars (RR lyrae, cepheids, Mira,...)

(g) Finally, after a long day stargazing, it's time to head back home. Your friends note that in the time you have spent there, the positions of the stars have shifted, except for Polaris, which still seems to be at the same position. You explain to them that this is because Polaris is a pole star. However, you also tell them that in two to three thousand years' time, Polaris will no longer be the pole star. What is the name of this phenomenon that causes this and why does it result in the pole star changing? **[2]**

Solution:

Precession of the equinoxes/axial precession. As the Earth precesses, the poles of the Earth will seem to trace out a circle over ~ 26000 years. This means that the north pole points to different stars over the course of the cycle, pointing at the same star again once every 26 000 years.

Question 2 How to land on the moon

Our obsession with visiting the Moon is timeless. It stretches as far back as ancient China, where the goddess Chang'e was supposedly to have drank an elixir of immortality and 'flown up to the Moon'.

In reality, one does not simply 'fly up to the Moon'. Complex calculations are needed to achieve such a feat. In fact, it was not until the middle part of the 20th century that it was attempted. As recently as 7th September 2019, India attempted a landing on the Moon. However, the attempt was unsuccessful for multiple reasons.

Part I Lift-off

Today, we shall attempt to land a probe of mass 1000 kg on the far side of the Moon. To achieve that, we first need to achieve lift-off.

(a) Calculate the minimum velocity the probe would need to obtain in order to escape from Earth's gravitational field. **[1]**

Solution:

For an probe to barely escape from the Earth's gravity, it need to be at the escape velocity v_{esc}which can be easily calculated by assuming the total energy of the probe is 0.Since the total energy of the probe is conserved during the flight to infinity, this would mean it reach infinity with no speed, thus leading to the minumium speed required on the Earth surface to escape its gravitational pull.

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_\oplus m}{R_\oplus} = 0 \quad (1)$$

$$v_{esc} = \sqrt{\frac{2GM_\oplus}{R_\oplus}} \quad (2)$$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.972 \times 10^{24}}{6378000}} \quad (3)$$

$$\approx 11200 m/s \quad (4)$$

Many rocket launchpads are constructed near Earth's equator, such as the one built in the French colony of French Guiana. This is usually done to conserve the amount of fuel needed to propel the rocket into orbit by taking advantage of the Earth's rotation about its own axis.

(b) Calculate the amount of energy saved by launching a rocket eastwards into a circular orbit with altitude 1000 km from the surface exactly at the Earth's equator, as compared to launching the same rocket to the same altirude from the surface from the South Pole. For this question, you may assume that the Earth is a perfect sphere.

(Note: You may assume for simplicity that the mass of the rocket does not change significantly. This however is not true in general, as fuel forms the bulk of a rocket's weight. Fuel is used to propel the rocket.[1]) **[2]**

Solution:

Earth's rotational speed can be calculated by considering that it rotates one round in 24 hours.

$$v_{earth} = \frac{2\pi R_\oplus}{T} \quad (5)$$

$$= \frac{2 \times \pi \times 6378000}{24 \times 60 \times 60} \quad (6)$$

$$\approx 463.8 m/s \quad (7)$$

[1]There are typically two types of rocket fuel used, namely, solid fuel and liquid fuel. For a rocket that goes to low Earth orbits, liquid hydrogen and liquid oxygen are typically used. Liquid oxygen acts as an oxidiser, which allows the fuel to burn in the absence of surrounding air. This type of fuel is often referred to as non-hypergolic fuel, which means that they do not spontaneously combust when the components come into contact. It is thus preferred in many situations, due to it being safer to handle.

To calculate the amount of energy saved, we make use of the fact that the total energy in orbit for both case are the same, thus the total energy saved should be equivalent to the amount of energy imparted into the probe due to the rotation of the earth which is given by:

$$\frac{1}{2}mv_{\text{earth}}^2 = \frac{1}{2} \times 1000 \times 463.8^2 \tag{8}$$

$$= 1.07 \times 10^8 J \tag{9}$$

(c) State Newton's three laws of motion, and explain how they apply to a rocket as it is launches upwards from the ground. Hence explain why, for a given magnitude of thrust, there is an upper limit on the payload that a rocket can carry. **[2]**

Solution:

Newton's First Law state's that an object stay in its state of motion unless acted on by a unbalanced force.This is applicable when the rocket is on the ground where the contact force with the ground balance out with the gravitational attraction of the earth.

Newton's Second Law states that the net force an object experience is related to the product of its mass and acceleration.This means that in order for the rocket to experience an net upward force, the rocket's thrust must be greater than gravitational attraction of the earth on the rocket.Since the gravitational attraction of the earth on the rocket is proportional to the mass of the rocket, thus this put an maximum payload mass on a rocket with a given thrust where the thrust is equal to the gravitational attraction of the earth on the rocket.

Newton's Third Laws states that there is always an equal and opposite force acting on a pair of body.In this case this explain the thrust generated by the engine since it eject particles as it combust, exerting an force on these particles and thus receive and equal and opposite force on the rocket known as thrust.

Part II Orbits

Magic happens, and our spacecraft has now achieved low Earth orbit at 1000 km. Also magically, we now live in the world of the 2030s and via a number of trans-lunar injections (TLIs)[2], the space station Lunar Orbital Platform-Gateway (LOP-G)[3] has been constructed and is now in lunar orbit. Our probe has thus magically teleported[4] to the LOP-G.

We now need to bring the spacecraft to the Moon. This is done via a transfer to the Hohmann orbit. Figure 1 shows a typical Hohmann transfer from a lower orbit (an orbit with a smaller semi-major axis) to a higher orbit (an orbit with a bigger semi-major axis). The middle transfer trajectory is known as a Hohmann transfer orbit.

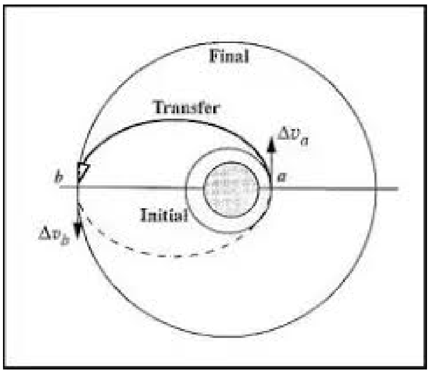


Figure 1: Illustration of a Hohmann transfer. Transfers can work in the other direction as well, from a higher orbit to a lower one.

The planned travel path for our probe to the surface of the Moon is as follows. The LOP-G will detach our probe when the LOP-G is aligned such that it is at perilune[5], on the near side of the moon opposite the intended landing site. The probe then performs a transfer to a Hohmann transfer orbit with perilune on the lunar surface, enabling the probe to land on the far side of the Moon.

Figure 2 shows this travel path. The LOP-G is[6] in an elliptical orbit with an apolune of 70000 km and a perilune of 3000 km.

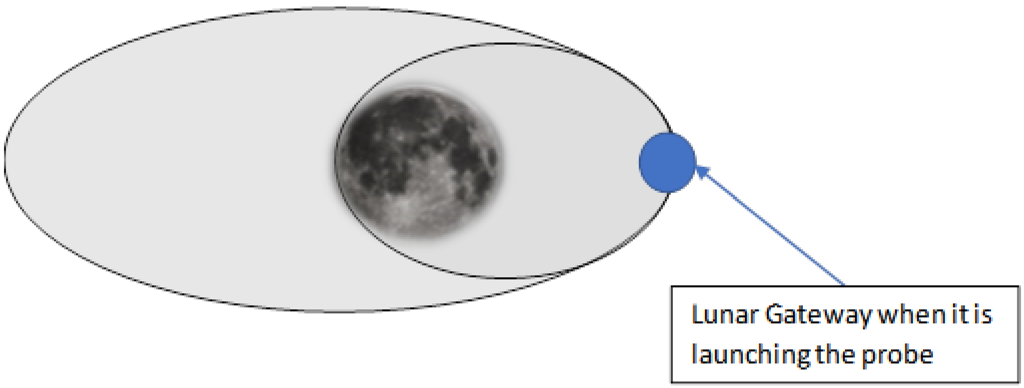


Figure 2: Illustration of the orbits of LOP-G and the probe, before and after the change in the probe's orbit.

(d) Estimate the flight time of the probe, from the time when it is with the LOP-G on the apolune of the LOP-G's orbit, to when it lands on the surface of the Moon. **[2]**

[2]A TLI is a propulsive manoeuvre approximating a Hohmann transfer. A Hohmann transfer brings a spacecraft from a lower orbit to a higher orbit, and is summarised in Figure 1. The difference between a TLI and a Hohmann transfer is that a Hohmann transfer recommends at least two major changes of velocity to change the orbit parameters, while the TLI only recommends one change of velocity.

[3]This is in fact a real planned space station with a planned launch in 2024. It will orbit the Moon.

[4]Read: It performed some amazingly well-calculated orbital transfers.

[5]Perilune is the point in the orbit closest to the moon. Similarly, apolune is the point in the orbit furthest from the moon.

[6]or is planned to be

Solution:

Using the Kepler's Third Law, we could get the period of LOP-G's orbit:

$$a_{\text{LOPG}} = \frac{70000 + 3000}{2} \quad (10)$$

$$= 36500km \quad (11)$$

$$P^2_{\text{LOPG}} = \frac{4\pi^2}{GM_{\text{moon}}} a^3_{\text{LOPG}} \quad (12)$$

$$P_{\text{LOPG}} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11} \times 7.348 \times 10^{22}} \times 36500000^3} \quad (13)$$

$$\approx 625852s \quad (14)$$

$$\approx 7.24\text{days} \quad (15)$$

Note:we could ignore the mass of the LOPG module and the probe since it is insignificant compare to the mass of the moon

The same hold for the probe's journey for to the moon surface, with the perilune being the moon's radius

$$a_{\text{probe}} = \frac{3000 + 1737.1}{2} \quad (16)$$

$$= 2368.55km \quad (17)$$

$$P^2_{\text{probe}} = \frac{4\pi^2}{GM_{\text{moon}}} a^3_{\text{LOPG}} \quad (18)$$

$$P_{\text{probe}} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11} \times 7.348 \times 10^{22}} \times 2368550^3} \quad (19)$$

$$\approx 10346s \quad (20)$$

$$\approx 0.120\text{days} \quad (21)$$

Thus the whole journey will take about $\frac{1}{2}(7.24 + 0.12) = 3.68$ days, answer in seconds are also accepted

(e) By using conservation of energy, derive an expression for the speed of the probe at any distance r away from the center of the moon when it is in a lunar orbit with a semi-major axis of a. **[3]**

Solution:

By using energy conservation condition, we can relate the distance r away from the center of the moon and its velocity v at this distance in an orbit with semi-major axis $a$by using the equation:

$$\frac{1}{2}mv^2 - \frac{GM_{\text{moon}}m}{r} = -\frac{GM_{\text{moon}}m}{2a} \quad (22)$$

$$v = \sqrt{GM_{\text{moon}}(\frac{2}{r} - \frac{1}{a})} \quad (23)$$

Note:solution similar to Option 2 of the next question will not be accepted since the momentum conservation condition used there is only true for apolune and perilune and does not apply in general.

(f) Hence or otherwise, calculate the change in velocity needed to get the probe to the surface of the Moon from the

LOP-G at the LOP-G's perilune via the orbit shown in Figure 2. You may assume the Moon is a perfect sphere. **[2]**

Solution:

There are two way to solve this question

Option 1

At the perilune, the velocity of the LOPG probe is given by:

$$v = \sqrt{6.67 \times 10^{-11} \times 7.348 \times 10^{22}(\frac{2}{3000000} - \frac{1}{36500000})} \quad (24)$$

$$\approx 1770.07 m/s \quad (25)$$

The speed of the orbit need to reach the moon's surface at this perilune distance of 3000km is given by

$$v = \sqrt{6.67 \times 10^{-11} \times 7.348 \times 10^{22}(\frac{2}{3000000} - \frac{1}{2368550})} \quad (26)$$

$$\approx 1094.61 m/s \quad (27)$$

Therefore, the change in velocity $\Delta v = 1770.07 - 1094.61 = 675.46 m/s$

Option 2

Using conservation of energy condition, we can obtain the relation between the speed v_1 of the probe at distance r_1 and the speed v_2 of the probe at distance r_2

$$\frac{1}{2}mv_1^2 - \frac{GM_{\text{moon}}m}{r_1} = \frac{1}{2}mv_2^2 - \frac{GM_{\text{moon}}m}{r_2} \quad (28)$$

We can also obtain an relation between the speed v_1, v_2, distance r_1,r_2 using the momentum conservation condition

$$mv_1r_1 = mv_2r_2 \quad (29)$$

$$v_2 = \frac{v_1r_1}{r_2} \quad (30)$$

Note:This relation is only true for velocity and distance at perilune and apolune and is not true in general due to the velocity and distance are not perpendicular at other points on the ellipse.

Substituing (30) into (28),we obtain the following

$$\frac{1}{2}mv_1^2 - \frac{GM_{\text{moon}}m}{r_1} = \frac{1}{2}m(\frac{v_1r_1}{r_2})^2 - \frac{GM_{\text{moon}}m}{r_2} \quad (31)$$

$$v_1^2(1 - \frac{r_1^2}{r_2^2}) = 2GM_{\text{moon}}(\frac{1}{r_1} - \frac{1}{r_2}) \quad (32)$$

$$v_1 = \sqrt{\frac{\frac{2GM_{\text{moon}}}{r_1^2}}{\frac{1}{r_1} + \frac{1}{r_2}}} \quad (33)$$

For the orbit of the LOPG at perilune, substitute r_1=3000km r_2=70000km/s into (33) and the speed at perilune is given by :

$$v_1 = \sqrt{\frac{\frac{2\times6.67\times10^{-11}\times7.348\times10^{22}}{r_1^2}}{\frac{1}{3000000} + \frac{1}{70000000}}} \quad (34)$$

$$= 1770.07 m/s \quad (35)$$

For the orbit of the probe at apolune, substitute r_1=3000km r_2=1737.1km/s into (33) and the speed at perilune is given by

$$v_1 = \sqrt{\frac{\frac{2\times 6.67\times 10^{-11}\times 7.348\times 10^{22}}{r_1^2}}{\frac{1}{3000000}+\frac{1}{1737100}}} \quad (36)$$

$$= 1770.07m/s \quad (37)$$

$$v_1 = \sqrt{\frac{\frac{2\times 6.67\times 10^{-11}\times 7.348\times 10^{22}}{r_1^2}}{\frac{1}{3000000}+\frac{1}{70000000}}} \quad (38)$$

$$= 1094.61m/s \quad (39)$$

The difference in the speed is the change in velocity needed which is about 675.46 m/s

(g) Hence or otherwise, determine the landing speed of the orbiter when it reaches the landing site, assuming no further actions are taken by the probe. **[1]**

Solution:

Similar to **e** there are two ways to solve this question

Option 1

Substition into (23) with r = 1737.1km and a = 2368.55km yields a landing speed given by :

$$v_{\text{landing}} = \sqrt{6.67\times 10^{-11}\times 7.348\times 10^{22}\left(\frac{2}{1737100}-\frac{1}{2368550}\right)} \quad (40)$$

$$= 1890.4m/s \quad (41)$$

Option 2

Substition into (33) with r_1 = 1737.1km and r_2 = 3000km yields a landing speed given by :

$$v_1 = \sqrt{\frac{\frac{2\times 6.67\times 10^{-11}\times 7.348\times 10^{22}}{r_1^2}}{\frac{1}{1737100}+\frac{1}{3000000}}} \quad (42)$$

$$= 1890.4m/s \quad (43)$$

Part III Communication

LOP-G, for magical reasons, seems to encounter a communications issue and therefore will not play any further role in this question. With the given set-up, the probe will have significant difficulty trying to reach and communicate directly with ground-based mission control.

(h) Explain why this is the case, and hence explain why the landing site is also a good location to place a radio telescope. **[1]**

Solution:

The moon will block any communications between the probe at the far side and the earth.

Hence it will also block out radio waves from the Earth. As such, the radio telescope would be free from interference.

The typical solution to the communication problem requires another satellite to act as a relay to communicate with Earth. To maintain constant communication with both the probe and the Earth, this satellite needs to be parked at a halo orbit[7] around a Lagrangian point, hereafter referred to as Location A, to maintain communication with both the Earth and the probe. One satellite already in this particular orbit is the Chinese satellite Queqiao, launched on 20th May 2018.

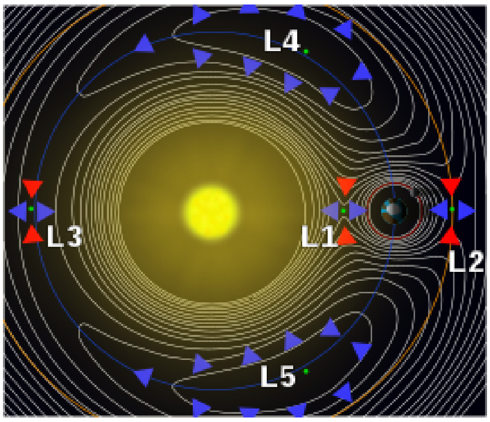


Figure 3: Illustration of all the Lagrangian points in the Sun-Earth system.

(j) Explain what is a Lagrangian point. **[1]**

Solution:

Lagrange points are positions in space where objects sent there tend to stay put.

OR

At Lagrangian points, the gravitational pull of two large masses precisely equals the centripetal force required for a small object to move with them

Both answer are accepted

(k) With the aid of a drawing of the Earth and the Moon, identify Location A. **[2]**

Solution:

The location in question is L2

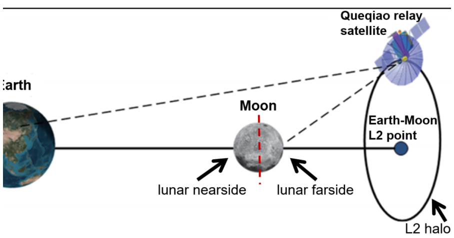


Figure 4: Illustration of the trajectory of communication satellite Queqiao

The relay satellite is able to maintain communication with the probe for most of the probe's duration on the Moon. However, it still encounters problem during occasions such as the Mid-Autumn Festival and during a solar flare/storm.

[7]A *halo orbit* is a periodic three-dimensional orbit around one of the L1, L2, or L3 Lagrangian points. It typically has a large orbital radius. The non-periodic variants of the same are called Lissajous orbits.

(m) The main issue for communication with the probe during the Mid-Autumn Festival[8] is that the probe's battery can potentially be too cold to operate. Explain why this can happen. **[1]**

Solution:

During a full moon event such as Mid-Autumn Festival, the far side of the moon could not receive enough sunlight as it was blocked by the near side of the moon.Furthermore due to the absence of an atmosphere on moon, the heat received on the near side of moon could not effectively transmit ot the far side.

(n) Solar flares and storms also pose a major issue to the inhabitants of the lunar-based probe. Suggest and explain one such issue. **[1]**

Solution:

Solar Flares contains highly energentic protons, these protons can penetrate the protection layers on the probe and cause serious damage to the onboard electronics. OR Solar Storm contain high energy photon such as X ray and gamma ray ,since they are highly ionizing radiations, they could post an potential radiation poisoning hazard to the onboard crews.

(o) Name the celestial phenomenon which you will likely see in the Northern hemisphere during a solar storm. Will you be able to see this phenomenon on the moon? **[1]**

Solution:

Aurora Borealis and no it does not occur on moon because it does not have a magnetic field

[8] Held on the 15^{th} day of the 8^{th} month of the Chinese lunisolar calendar, when the Moon is brightest and at its full size.

Question 3 Bloom and Boom

Introduction

When it comes down to the wire, what the Universe consists of is really a cycle of things blooming, then going boom. First, the biggest boom and bloom of all, the Big Bang. Then, stars and galaxies bloomed. At the end of their life cycles, loads of stars go boom. Material is expelled, and new stars eventually bloom. Rinse and repeat.

Part I The Biggest B(l)oom

As you should know, the Big Bang is the commonly accepted model that explains the origins of our Universe today. There are four main pillars of evidence to support the Big Bang hypothesis. In no particular order, they are:

- expansion of the Universe,
- cosmic microwave background radiation,
- nucleosynthesis of light elements, and
- formation of galaxies and the large-scale structure of the Universe.

(a) Choose two of the four pillars and give a short explanation about how they support the Big Bang hypothesis. **[2]**

Solution:

Any two of the following:

- Expansion: The Universe is expanding at all points in all directions as per Hubble's Law. This is a consequence of the initial explosion.
- CMBR: After the recombination era (formation of hydrogen from free protons and elec- trons), photons are able to travel through the Universe freely (i.e. transparent to ra- diation). The CMBR today permeates the entire Universe and is this free radiation, redshifted.
- Nucleosynthesis: The proportion of light elements (for example Helium 4) observed is in good agreement with theoretical calculations of the nuclear processes in the 'hot phase' of the Big Bang.
- Formation: As temperature falls after the Big Bang, the main component of energy density of the Universe switched from radiation to particulate in nature. This allowed small per- tubations in matter density to grow via gravitational interactions amongst more massive particles, forming the structure today.

Part II Blooming Stars

Of the many stars that have bloomed, a certain type of star has been crucial in our astronomical studies, allowing us to determine distances. These are, of course, variable stars. The common examples are Cepheids and RR Lyrae variables, and they are often one of the first things to come to mind when the phrase 'standard candle' is mentioned.

What makes these stars immensely useful is their well-defined period-luminosity relationship. Variable stars 'bloom' at very regular intervals – their luminosities increase and decrease with a fairly constant period. This period is well-correlated with their absolute luminosity, allowing us to determine their distance from us (even though one should be aware of calibration difficulties due to effects of metallicity and/or blending).

The following table provides some data of typical Cepheids and RR Lyrae variables.

Property	Type I Cepheids	Type II Cepheids	RR Lyrae
Composition	Metal-rich	Metal-poor	Metal-rich
Period	Days to months	1 to 50 days	Hours
Age	Young	Old ($\approx 10\,\text{Gyr}$)	Old
Mass	Greater than $2M_\odot$ or $3M_\odot$	Low	Low ($\approx 0.5M_\odot$)

Table 1: Some characteristics of variable stars.

(b) The mechanism of 'blooming' of Cepheids and RR Lyrae variables are mostly identical. In fact, two meanings of 'blooming' can be said to take place. The star blooms in size. Then the star blooms in luminosity. Then it contracts, returning to a smaller size with lower luminosity. Then it blooms in size again.

The mediating agent, as it were, is thought to be an external shell of ionised helium. Doubly ionised helium has higher opacity to radiation as compared to singly ionised helium. Given this information, explain the mechanism of 'blooming' of Cepheids and RR Lyrae variables. You should account for both 'blooming's in your explanation. **[3]**

Solution:

The mechanism relies on the heating and cooling of the helium envelope, mainly.

When the helium envelope is compressed, it heats up. Doing so converts the singly ionised helium in the envelope to doubly ionised helium. The doubly ionised helium is opaque to radiation, trapping radiation between the envelope and the star. This also causes a drop in luminosity, since photons cannot escape the envelope.

The pressure build-up causes the helium envelope to expand. As it does so, the envelope cools, reverting doubly ionised helium to singly ionised. This allows photons to escape, causing a reduction in pressure and an increase in luminosity. The reduction in pressure causes the envelope to compress again due to gravity overcoming radiative pressure. This sets up the cycle to repeat again, as the envelope once again heats up under compression.

It is a fact that different standard candles are used for different distance measurements in the Universe. A standard candle used for nearer distances may not work for larger distances, and vice-versa. This concept is frequently called the *distance ladder*.

(c) The two Cepheid variables are used to determine distances of different objects. With reference to the given table, or otherwise, suggest objects each Cepheid variable is used to determine distances of. **[1]**

The period-luminosity relations for both Cepheid types are as follows.

Star	Relationship
Type I	$M_v = (-2.43 \pm 0.12)(\log_{10} P - 1) - (4.05 \pm 0.02)$
Type II	$M_v = -2.81(\log_{10} P - 1) - 2.66$

Table 2: Period-luminosity relations for the two main Cepheid types. Here, M_v is the mean absolute magnitude, and P is the period of pulsation in days.

Solution:

Type I: Galaxies (including extragalatic distances), especially star-forming galaxies.

Type II: Globular clusters, metal-poor/old galaxies, galactic centre. In essence, anything with old/population II stars.

The following is light curve data from a nearby quadruple system Cepheid star δ Cephei (specifically from the variable star δ Cephei A), whose observed light suffers from 0.23 magnitude extinction due to interstellar dust. Its pulsation period is 5.366249 days. It is thought to be about 4.5 solar masses and about 100 million years old.

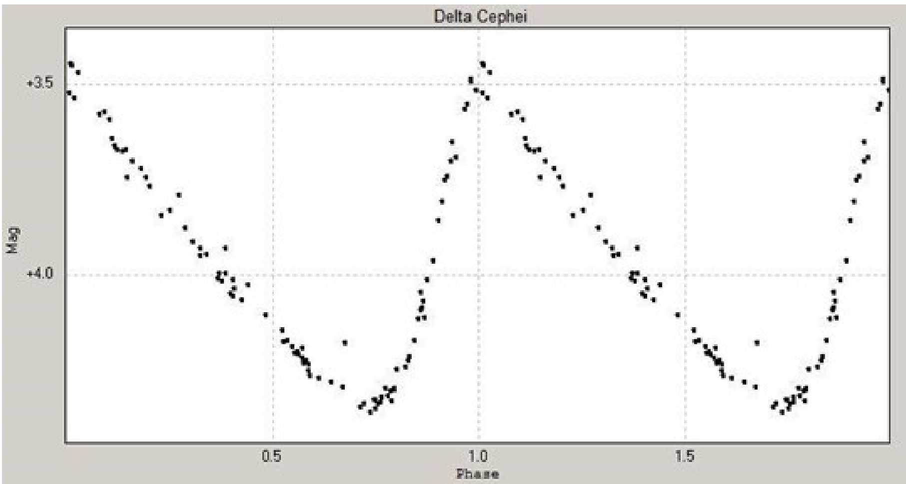


Figure 5: Observed light curve of δ Cephei.

(d) Determine the approximate distance of δ Cephei from Earth. **[3]**

Solution:

From the given information, it is inferred that δ Cephei is a Type I Cepheid variable. Ignoring the uncertainties in the relationship equation,

$$M_v = -2.43(\log_{10} 5.366249 - 1) - 4.05 \quad (44)$$
$$= -3.39310. \quad (45)$$

One now observes the mean apparent magnitude to be approximately +3.9. Taking into account extinction, we expect the mean apparent magnitude, factoring into account extinction, to be approximately +3.67. By formula booklet,

$$3.67 - (-3.39310) = 5\log_{10}\frac{d}{10} \quad (46)$$

which gives

$$d = 10^{2.41262} \quad (47)$$
$$= 258.59493 pc \quad (48)$$
$$= 843 ly. \quad (49)$$

Note: Strictly speaking, one should convert the data to intensity units, then do some fancy integration to get the BVI mean magnitude. But we can't do that here, so we settle for the next best thing, an averaging.

Part III Booming Stars

One of the other well-known standard candles are Type Ia supernovae.

(e) Type Ia supernovae are used to provide evidence for one of the four pillars of the Big Bang hypothesis. Which one, and how? **[1½]**

Solution:

Expansion of the universe. Type Ia supernovae have well-studied light curves and nearly identical intrinsic brightness. Therefore we can infer distance from the brightness. Measuring the cosmological redshift and correlating with distance yields the needed empirical evidence.

As is well-known, at the end of their lives, stars have one of three main fates. They can boom into a supernova, they can b(l)oom into a planetary nebula, or they can (hypothetically) just not boom/bloom into anything, kind of like a really sad flower bud.

(f) What is the difference between a supernova and a planetary nebula, in terms of their cause/formation? **[1]**

Solution:

Supernovae are stellar explosions. They are caused following a sudden core collapse when the core becomes too heavy for degeneracy pressure to support the gravitational pull.

(Note: We acknowledge the difference between Type I and II supernovae. While it is true that the mechanisms for booming are different, they have the same principle, that of core collapse.)

The following are pretty pictures of planetary nebulae. Note the pretty petal-like structures, pointed out with arrows, almost like the planetary nebulae are blooming.

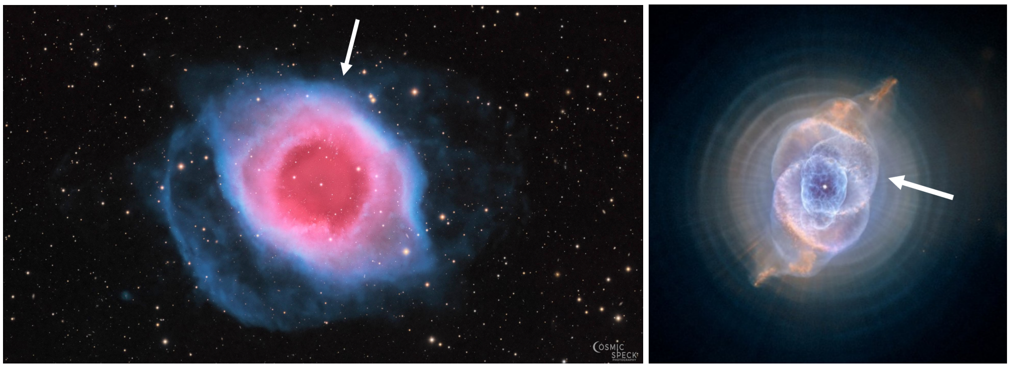


Figure 6: Two famous planetary nebulae.

(g) Explain the cause of these structures in detail. **[2]**

Solution:

At the end of a intermediate mass star's life (and after it has expanded into a red giant), it eventually runs out of hydrogen/helium for nuclear fusion. The core temperature is insufficient to trigger the CNO cycle. When this occurs, there is a sudden drop in radiative pressure, i.e. the force of gravity overpowers radiative pressure and causes a sudden rapid stellar contraction.

When this happens, stellar temperatures rise rapidly again. Nuclear processes in the outer shells (helium fusion) take place, causing another rise in radiative pressure outward. However, gravity is insufficient to overcome this pressure, causing the outer layers of the star to be shed. The shed outer layers expand and form these bloomin' pretty structures.

In general, it is difficult to determine distances of planetary nebulae from Earth. However, for short distances (approximately $< 1000\,\mathrm{kpc}$), it is possible to directly (visually) resolve sufficiently large nebulae and perform a distance measurement by comparing measurements several years apart.

(h) Propose a method to perform this distance measurement. **[2]**

Solution:

We need two pieces of information.

- A visual measurement of expansion (size) several years apart. This shows expansion perpendicular to line of sight.
- A spectroscopic measurement. This shows Doppler shifts along the line of sight.

The spectroscopic measurement allows us to determine velocity of expansion. By this we can determine from the velocity of expansion and the time elapsed, the total distance of expansion. From this and the difference in angular sizes of the nebula over the different observations years apart, we can find the distance to the nebula itself.

Part IV B(l)oom Into You

No question about blooming and booming is complete without discussing the most important bloom and boom: that which became you! The bloom and boom of life is a widely-discussed topic in both astrobiology and in science fiction: just look at Star Trek, or Doctor Who, or even any number of Japanese animated series.

Many portrayals of extraterrestrial life in science fiction takes place on planets where water is present. Often, this goes hand-in-hand with some mention of the 'Goldilocks zone', properly known as the *Circumstellar Habitable Zone*

(CHZ). This reliance on water is not unique to science fiction, however: much of the effort made by astronomers today in finding extraterrestrial life focuses on planets and moons where water could be present.

(j) Define the term *Circumstellar Habitable Zone*. **[1]**

Solution:

The CHZ is the range of orbits about a star (or star system, e.g. in the case of binaries etc.) within which a planet's surface can support liquid water, given sufficient atmospheric pressure.

(k) The CHZ is a crucial concept in the search for extraterrestrial life. Explain why. **[1½]**

Solution:

Life as we know it is water-based. Much of Earth-like life and biosphere depends on water. Critically.

Due to this, the nature of the CHZ and objects within are important in determining the possible range of planets able to support Earth-like life.

(m) Despite its well-known status, relying solely on determining the CHZ is not exactly a reliable method of finding potentially habitable worlds. Suggest and explain briefly two reasons why. **[2]**

Solution:

Choose any two from the below. Other reasonable answers are welcome.

- The CHZ does not take into account stellar activity, such as flares. Flares are deadly without proper shielding, and could potentially hinder survivability of life.
- The CHZ does not account for the UV habitability zone. With, for example, high UV in the CHZ, liquid water undergoes photoevaporation in the formative stages, leading to a lack of water.
- The CHZ does not account for subsurface oceans outside the zone. Such oceans potentially contain life as well.
- The CHZ does not account for other sources of heating outside the star. For example, internal geology or tidal heating could potentially warm a body sufficiently outside the CHZ.
- The CHZ potentially migrates over time. This is especially significant for high-mass stars which burn and change stages relatively quickly, such as O class stars. If the CHZ keeps shifting, a planet may not stay within the CHZ long enough for life to develop.
- The CHZ is notoriously difficult to calculate, as it relies on a number of different factors such as spectral class and star type and activity.
- Extraterrestrial life may not be found 'as we know it'. For example, life elsewhere could be silicon-based, as compared to carbon-based on Earth.

Question 4 Hero among the constellations – Orion

Orion is perhaps one of the most famous constellations in the night sky due to its many bright stars and iconic shape. From tales of his rabbit hunt with his hunting dogs to his death upon facing a giant scorpion, Orion's legend too remains in the night sky as one of our most iconic myths.

To introduce the stars and lesser-known stories about Orion, you will have to overcome five trials and five poems about the ancient hero. Are you ready?

Common name	Bayer designation	Star type (HR, MK system)[9]	Distance (light years)	Apparent magnitude			Machanism for Variability
				Average	Max	Min	
Betelgeuse	α Orionis	M1-M2 Ia-ab	624	0.42	0	1.62[10]	Semi-regular Variable
Rigel	β Orionis	B8 Ia	772	0.12	0.05	0.18	α Cygni Variable
Bellatrix	γ Orionis	B2 III	245	1.62	1.59	1.64	Suspected Variability
Mintaka	δ Orionis	(O9.5 II+B1 V +B0 IV)+B3 V	916	2.26	2.23	2.29	Eclipsing Binary with Triple Star System
Alnilam	ϵ Orionis	B0 Ia	1342	1.69	1.64	1.74	α Cygni Variable
Alnitak	ζ Orionis	O9.5 Iab +B1 IV+B0 III	800	1.77	NA	NA	Triple Star System
Saiph	κ Orionis	B0.5 Ia	???	2.07	2.05	2.09	Suspected Variability

Table 3: Summarised data for several bright star in the constellation of Orion

Part I Betelgeuse

"A red beetle crawled under his arm and was crushed into beetle juice.
Immortalised in the night sky, we now call it Betelgeuse."[11]

Betelgeuse is a semiregular variable star.

(a) Calculate how many times brighter Betelgeuse is at its maximum compared to its record low earlier this year. **[1]**

Solution:

The ratio between the maximum and minimum luminosity can be calculated using the formula book to give:

$$\frac{L_{\max}}{L_{\min}} = 10^{\left(\frac{1.62-0}{2.5}\right)} \tag{50}$$

$$\approx 4.446 \tag{51}$$

(b) Suggest why stars like Betelgeuse are not a good candidate for standard candles. **[1]**

[1]As classified on the Hertzsprung-Russell diagram, under the Morgan-Keenan system.
[2]New record set in February 2020
[11]Not actually how 'Betelgeuse' is derived.

Solution:

The variability of Betelgeuse is irregular and does not act as a mechanism that can regularly/ reliably/ accurately predict its actual luminosity, a feature that is important for astronomical standard candles (for measuring its distance).

Part II Rigel

"Putting his best foot forward, Orion stands, proud and regal.
Though but the sandal of a great warrior, people name it Rigel."[12]

(c) What is an asterism? **[1]**

Solution:

A**prominent group of stars** that are officially recognised to form a **distinct pattern** outside the scope of recognised constellations.

(d) Which famous asterism is Rigel in? Name one other star in the same asterism. **[2]**

Solution:

Winter hexagon.

Aldebaran, Capella, Pollux, Procyon, Sirius.

Accept Castor and Betelgeuse (some sources include them) only if the winter Hexagon is identified correctly.

A student new to astronomy saw Rigel's designation as a B8 Ia star. However, Rigel will end its life cycle as a Type II supernova, not a Type Ia supernova.

(e) Explain briefly why this is the case. **[2]**

Solution:

Rigel is a blue supergiant and **exceeds the Chandrasekar limit.**

(Explanations that points to the mechanism behind the Chandrasekar limit or elaborate on it is welcome, though recognising the above is enough. Subtract marks if 'smoking' or lack of understanding is present.)

Bonus mark:

B8 Ia refers to spectral type and **H-R classification** as an Ia Supergiant - not to be confused with type Ia supernovas, which refers to the light curve designation for supernovae. (1m if answer displays clarity in this regard)

Ia stars do not necessarily undergo Type Ia supernovae (0.5m for reasoning along these lines, but without clear understanding).

Part III Bellatrix

"On his other shoulder plate is a portrait of his beloved.
The Amazoness star Bellatrix."[13]

(f) Calculate the average absolute magnitude of Bellatrix. **[2]**

Solution:

Using the distance modulus formula in the formula book gives

$$1.62 - M = 5\log_{10}\frac{75.1173pc}{10pc} \tag{52}$$

$$M = -2.7587 \tag{53}$$

[12] Not actually how 'Rigel' is derived.
[13] Not actually how 'Bellatrix' is derived.

Bellatrix is approximately 6 times the size of the Sun in diameter, and is approximately 9000 times more luminous than the Sun.

(g) Assuming Bellatrix is a main sequence star, calculate the surface temperature of Bellatrix using Stefan-Boltzmann law. **[2]**

Solution:

substituting information for Bellatrix in to the Stefan-Boltzmann Law gives:

$$9000L_{\odot} = 4\pi(6R_{\odot})^2\sigma T^4 \tag{54}$$

$$T = \sqrt[4]{\frac{9000 \times 3.828^{26}}{4\pi \times (6 \times 696340000)^2 \times 5.67 \times 10^{-8}}} \tag{55}$$

$$\approx 22938.4K \tag{56}$$

(h) Comment briefly on whether your result in part **g** makes sense, given Bellatrix's classification as a B2 III giant. **[2]**

Solution:

If answer is correct: Yes.

Value is in the right ballpark on the H-R diagram (Appendix A); Or, show by calculation/ example/ comparison with another known star, e.g. the Sun; any reasonable approach is accepted.

If answer is wrong: No.

Value is in wrong ballpark on H-R diagram/ candidate is able to point out the absurdity of the value; will give full credit + method marks for above if answered well

Part IV Belt stars

"On his belt, three bright stars to the common eye line up.
Mintaka, the belt buckle;
Alnilam, where his great scabbard and sword hangs;
Alnitak, where he keeps a flame in a lit lantern and a horsehead chess piece."[14]

Apart from the Orion Nebula (M42), the Flame Nebula (NGC 2024) and Horsehead Nebula (Barnard 33) are two other notable nebulae also found in Orion.

Figure 7: Image of Flame Nebula (left) and Horsehead Nebula(right)

The Flame Nebula appears reddish-yellow when imaged in visible light. On the other hand, it is extremely difficult to image the Horsehead Nebula, which appears as a black horsehead against a reddish backdrop.

(j) What type(s) of nebulae are the Flame Nebula and Horsehead Nebula respectively? **[1]**

> **Solution:**
>
> Emission/ Dark Nebula respectively.
>
> Accept if Horsehead is stated to be Dark + Emission (0.5 marks if only Emission is mentioned)

(k) For each of the two nebulae, state whether they are expected to have star-forming regions. Justify. **[2]**

> **Solution:**
>
> Both of them.
>
> The Flame nebula is comprised of ionized hydrogen lit by a newly formed open cluster of young stars.
>
> The Horsehead nebula is a cloud of dense, opaque interstellar cloud where star formation is still ongoing.

(m) Mintaka and Alnitak are both multi-star systems. Why does Mintaka show variability in brightness, but Alnitak does not? **[1]**

> **Solution:**
>
> Our view of the orbital plane of Mintaka is aligned such that two stars block each other, but not for Alnitak.
>
> Accept: Part of Mintaka is an eclipsing binary, which involves celestial bodies that block each other and would cause it to appear less bright.

[14] Not actually how 'Mintaka', 'Alnilam', and 'Alnitak' are derived.

Part V Saiph

"His back foot, anchored against the ground, keeps the hunter safe.
Proud and strong Orion poses, the other sandal thus named Saiph."[15]

The absolute magnitude of Saiph is −6.1.

(n) Based on this, calculate Saiph's distance from us in light years.

(Note: You may find additional data from the table above, as well as the formula book, useful.) **[3]**

Solution:

Using the distance modulus formula from formula booklet and the apparent modulus of Saiph is +2.07.

$$2.07 - (-6.1) = 5\log_{10}\frac{d}{10pc} \tag{57}$$

$$d = 10^{\frac{8.17}{5}} \times 10pc \tag{58}$$

$$= 430.5pc \tag{59}$$

$$\approx 1404ly \tag{60}$$

[15] Not actually how 'Saiph' is derived.

Question 5 'Ike Mākou i Ke Ala

The ancient Polynesians were known to be remarkable navigators. Their canoes sailed on thousands of kilometres of open ocean to bring settlement to islands across the Pacific, from Hawaii, to Rapanui (Easter Island), to Aotearoa (New Zealand). Western navigational instruments such as compasses, clocks, and sextants were not available to the ancient Polynesians; they instead relied on nature-based navigational indicators like currents, clouds, bird movements, and at night when these things were nearly impossible to see stars. Imaginary lines were traced to connect stars in the night sky to help Polynesian voyagers memorise where and when different stars rise and set as well as the paths these stars would take throughout the night.

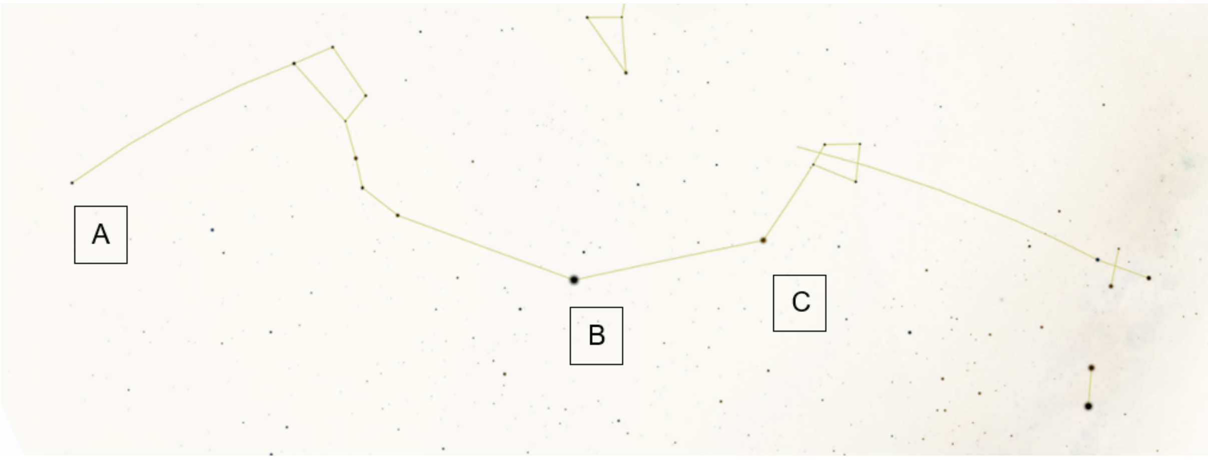


Figure 8

(a) One of these constellations is depicted below in Figure 8. State the modern name for each labelled star in parts i to iii.

(i) Star A. **[1]**

Solution:

Polaris

(ii) Star B. **[1]**

Solution:

Arcturus

(iii) Star C. **[1]**

Solution:

Spica

(b) Figure 9 shows the Navigator's Triangle on an equatorial grid. They are visible for most of the night; hence they are frequently used as markers for navigation.

(i) Three stars form the vertices of the Navigator's Triangle (Stars D/E/F in Figure 9). Name the modern IAU constellations that contain these three stars. **[1½]**

Solution:

Here are the morden constellation inside the Summer Triangle

- Cynus

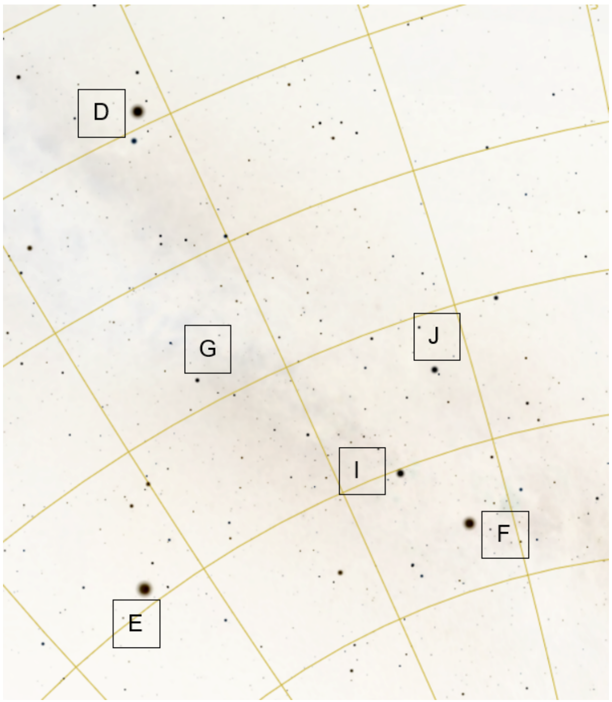


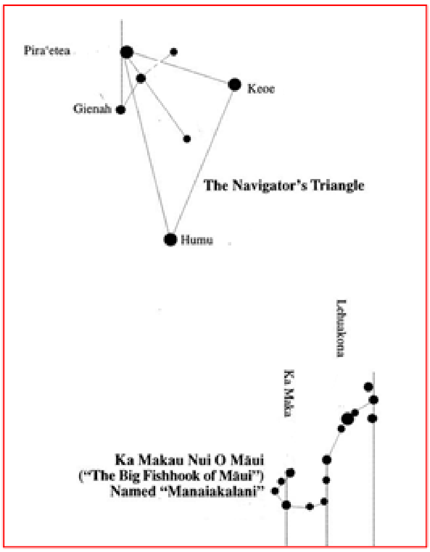
Figure 9

- Lyra
- Aquila

(ii) Demonstrate how the ancient Polynesian explorers possibly used the Navigator's Triangle to determine the direction of North. Two of the stars in the region of the Navigator's Triangle were particularly useful for them. You should use Figure 9 to aid in your explanation. [1½]

Solution:

From the preamble of Question Huinakolu is North from Manaiakalani hence North is somewhere vertically upwards from Manaiakalani. Students can identify Gienah and Pira'etea as stars of Cygnus (With Pira'etea being Denebola). At the orientation given above, this would mean that a line drawn from Gienah and Pira'etea would point towards the North Meridian. Refer to the diagram below to see the line from Gienah to Pira'etea.



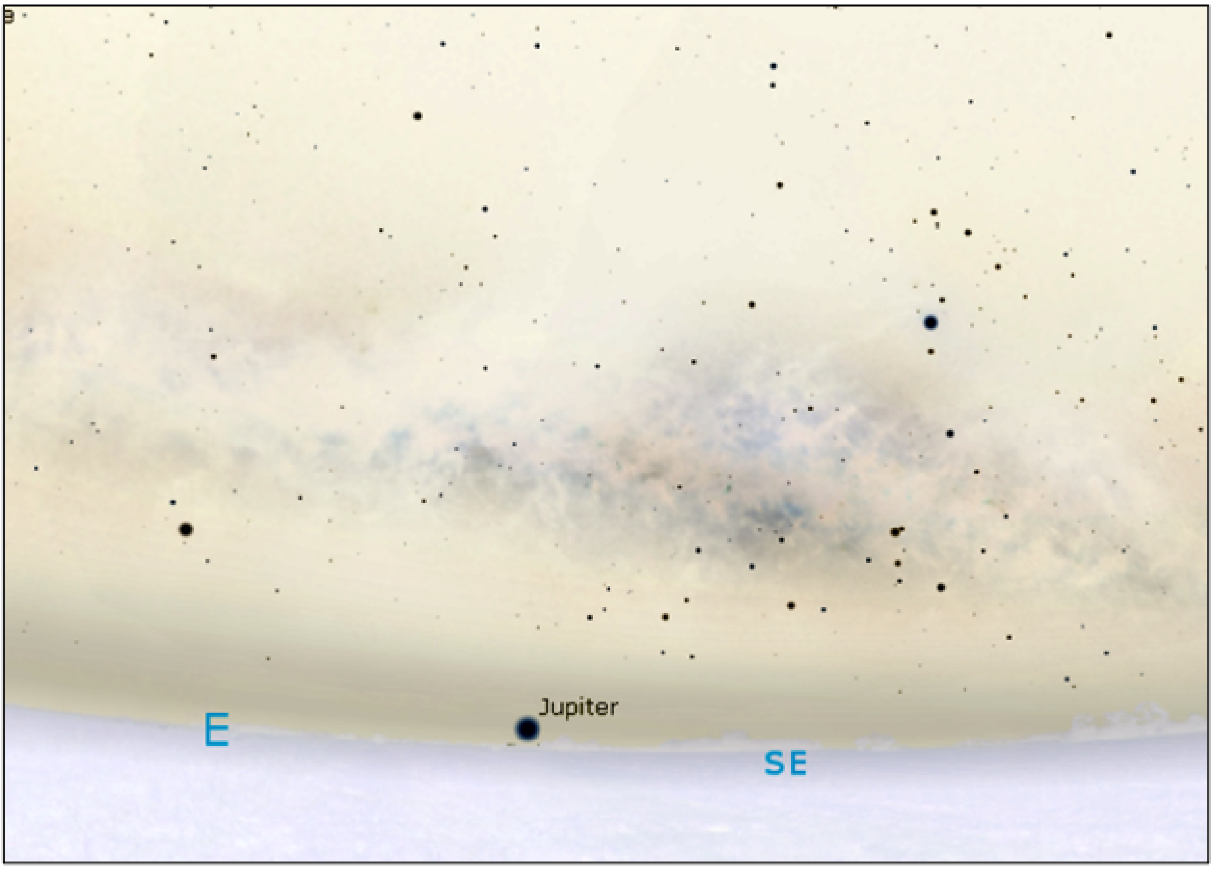
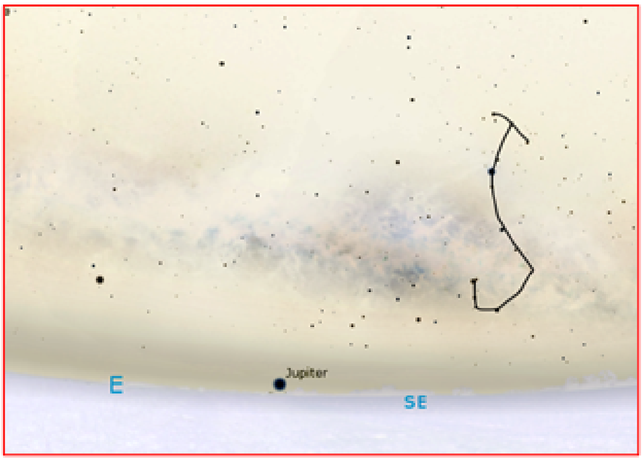


Figure 10

(c) The Chief's Fishing Hook is shaped like a fishhook lodged into a dark area of the Milky Way, poetically termed as the great celestial fish. Figure 10 is a picture of the fishhook and great celestial fish as they rise from the Eastern horizon.

(i) On Figure 10, trace the constellation of the Chief's Fishing Hook. **[1]**

Solution:



(ii) The Chief's Fishing Hook coincidentally shares the same stars as a major IAU constellation. What constellation is this and what is the modern name of its brightest star? **[2]**

Solution:

Scorpius, Antares

(iii) In the Hawaiian language there is a poetic phrase to describe the movement of the great celestial fish across the night sky: 'Ua huli ka I'a', meaning 'the fish has turned'. What astronomical phenomenon of the Milky Way could this phrase describe? Explain your answer. It may be helpful to consider how the fish hook and fish look like when they set into the Western horizon. **[2]**

Solution:

The Milky Way transits the Meridian and starts to set in the West.

As the fishhook and fish rise from the Eastern horizon the fishhook appears to be pulling the fish out of the horizon and when they set into the Western horizon the fishhook appears to be pulling the fish down into the horizon.

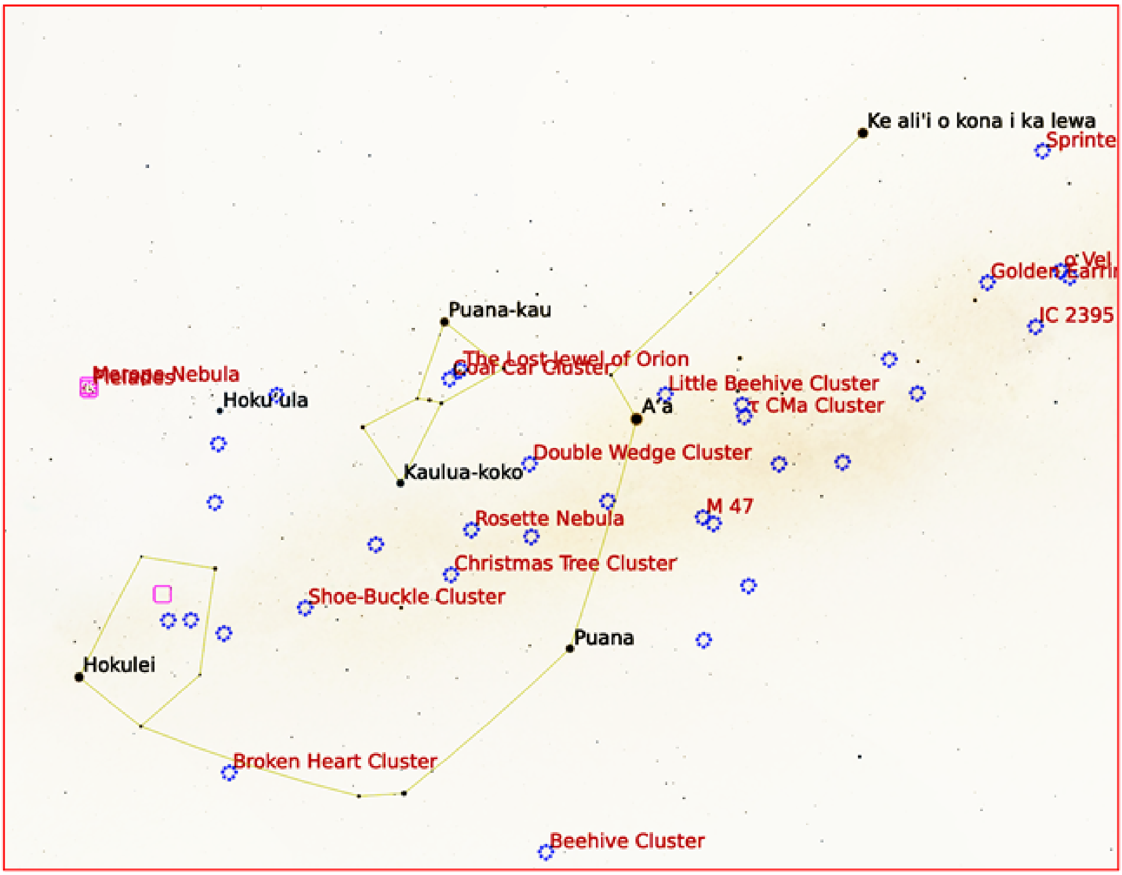
Figure 11

What might be poorly translated into English as a "Scoop" is visible throughout most of the night during winter and is frequently used as a marker by the ancient Polynesians. Many deep sky objects (DSOs) that are visible even in Singapore can be seen in or around the Scoop. Figure 11 is a field of view of the night sky containing the Scoop. The constellation has already been drawn out.

(d) Name and mark at least **four** DSOs in Figure 11. **[2]**

Solution:

below is the non exhausting list of DSO within the given field of view.



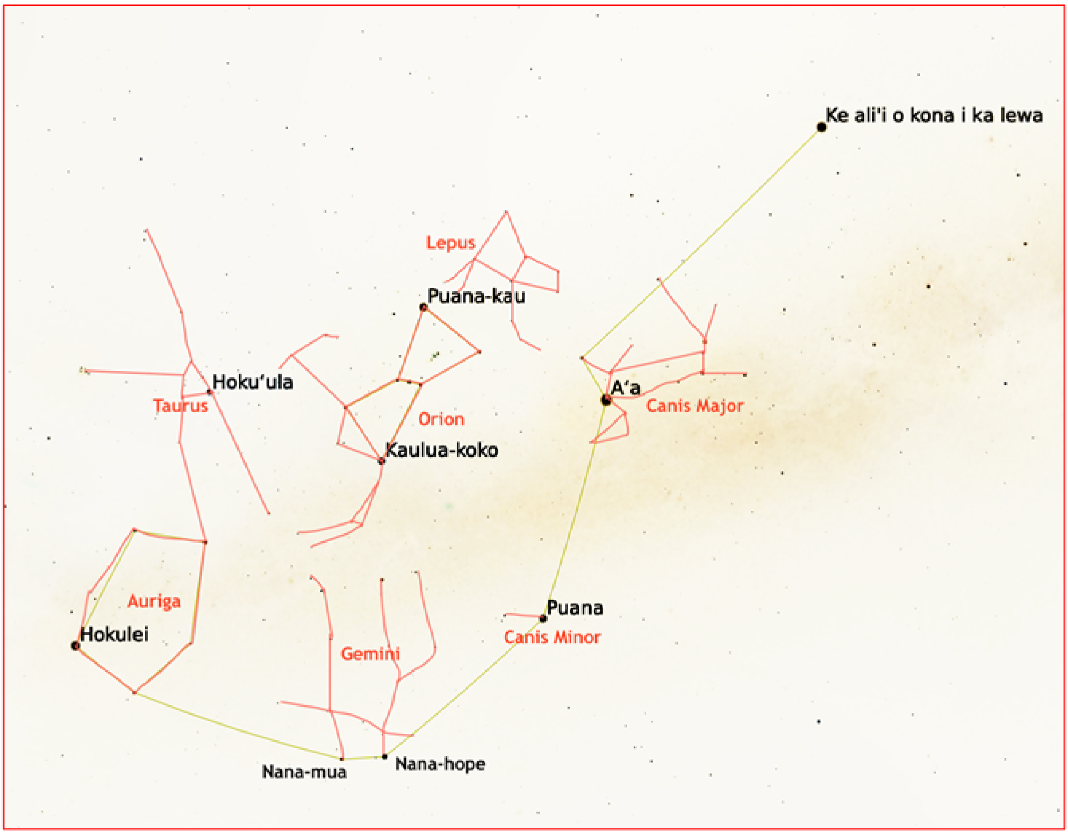
[0.5m for every DSO marked and named correctly]

(e) The Scoop can be seen over Singapore in the winter months. Thus, it is home to many winter constellations and two well-known winter asterisms from the modern star catalogue.

(i) Also within Figure 11, name and trace at least **three** modern constellations that appear in the field of view given of the Scoop. **[3]**

Solution:

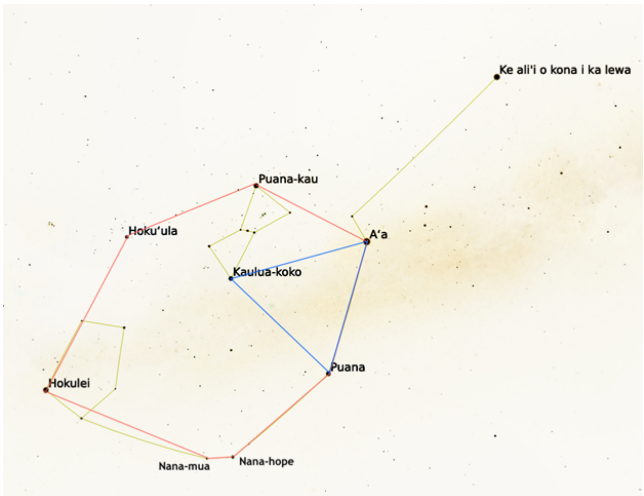
Any of the constellation in given in the image below are correct.



[1m for each constellation]

(ii) Name and trace the **two** modern winter asterisms in Figure 11. **[2]**

Solution:



[0.5m for tracing each asterism]

Winter Hexagon (in red) [0.5m], Winter Triangle (in blue). [0.5m]

(f) Refer to Figure 11 to answer the following parts **i** and**ii**. State the modern names of the celestial objects given.

(i) Star K **[1]**

Solution:

Sirius

(ii) Star L **[1]**

Solution:

Aldebaran.