AstroChallenge Formula Booklet

revised 30th May 2024

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This booklet consists of 5 printed pages, excluding this cover page.
2. Do **NOT** make any markings on this booklet.
3. Return this booklet to the invigilator at the end of this round of competition together with your answer script.

1 Useful Constants

Table 1: Physical and orbital characteristics of selected bodies in the Solar System

Property	Sun ☉	Mercury ☿	Venus ♀	Earth ♁	Moon ☾	Mars ♂	Jupiter ♃	Saturn ♄	Uranus ⛢	Neptune ♆
Mass m / kg	1.989×10^{30}	3.302×10^{23}	4.868×10^{24}	5.972×10^{24}	7.348×10^{22}	6.419×10^{23}	1.899×10^{27}	5.685×10^{26}	8.681×10^{25}	1.024×10^{26}
Radius R / m	6.963×10^{8}	2.439×10^{6}	6.051×10^{6}	6.370×10^{6}	1.738×10^{6}	3.396×10^{6}	7.149×10^{7}	6.027×10^{7}	2.556×10^{7}	2.476×10^{7}
Orbital semi-major axis a / m	-	5.791×10^{10}	1.082×10^{11}	1.496×10^{11}	3.843×10^{8}	2.279×10^{11}	7.785×10^{11}	1.433×10^{12}	2.877×10^{12}	4.503×10^{12}
Orbital period T	-	87.97 days	224.70 days	365.24 days	27.322 days (sidereal) 29.531 days (synodic)	686.97 days	11.86 years	29.46 years	84.32 years	164.79 years
Orbital eccentricity ϵ	-	0.205	0.0067	0.0167	0.0549	0.0933	0.0488	0.0557	0.0444	0.0112

Table 2: Commonly used fundamental constants and unit definitions

Units and Physical Quantities	Universal Constants
1 astronomical unit (AU) $= 1.49597870700 \times 10^{11}$ m	Planck's constant $h = 6.62606957 \times 10^{-34}$ m^2 kg s^{-1}
1 light year (ly) $= c \times 1$ year $\sim 9.4605284 \times 10^{15}$ m	Reduced Planck's constant $\hbar = \frac{h}{2\pi}$
1 parsec (pc) ≈ 3.26163344 ly	Gravitational constant $G = 6.67384 \times 10^{-11}$ N m^2 kg^{-2}
1 electron-volt (eV) $\approx 1.60217657 \times 10^{-19}$ J	Speed of light $c = 2.99792458 \times 10^{8}$ m s^{-1}
Avogadro's number, $N_A = 6.0221413 \times 10^{23}$	Boltzmann's constant $k_B = 1.3806488 \times 10^{-23}$ J K^{-1}
Average solar luminosity, $L_\odot \approx 3.846 \times 10^{26}$ W	Coulomb constant $k_e = \frac{1}{4\pi\epsilon_0} = 8.98755179 \times 10^{9}$ N m^2 C^{-2}
Average solar temperature, $T_\odot \approx 5778$ K	Stefan–Boltzmann constant $\sigma \approx 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Atomic mass unit $u \approx 1.660539 \times 10^{-27}$ kg	Electronic charge $q_e \approx 1.602 \times 10^{-19}$ C
Proton mass, $m_p \approx 1.672622 \times 10^{-27}$ kg $= 1.007276u$	Fine structure constant $\alpha = \frac{k_e (q_e)^2}{\hbar c} \approx \frac{1}{137}$
Neutron mass, $m_n \approx 1.674927 \times 10^{-27}$ kg $= 1.008665u$	Wien's displacement constant $b \approx 2.89776829 \times 10^{-3}$ m K
Electron mass, $m_e \approx 9.10938 \times 10^{-31}$ kg	Hubble constant $H_0 = 67.80 \pm 0.77$ km s^{-1} Mpc^{-1} (as of March 2013)

2 Useful Formulae

Table 3: Mathematical formulae

Description	Formula
Arc length on a circle is proportional to circular angle in radians	$s = r\theta$ (Gives the circumference when $\theta = 2\pi$)
Law of sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2R$ (on a plane) $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$ (on a sphere)
Law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$ (on a plane) $\cos c = \cos a \cos b + \sin a \sin b \cos C$ (on a sphere)
Small-angle approximations ($x \ll 1$, x in radians)	$\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$
First-order binomial expansion	$(1+x)^y \approx 1 + xy$

Table 4: Classical Astrophysics

Description	Formula
Kinetic energy	$E_{\text{kin}} = \frac{1}{2}mv^2$
Newton's universal law of gravitation	$\vec{\mathbf{F}} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$
Gravitational potential energy	$E_{\text{pot}} = -\frac{Gm_1m_2}{r}$
Gravitational binding energy of a uniform sphere	$U = -\frac{3}{5}\frac{GM^2}{R}$
Roche limit for a small, rigid body of density ρ_2 approaching a larger body of density ρ_1 and radius R	$d_{\text{Roche}} = 1.26R \times \left(\frac{\rho_1}{\rho_2}\right)^{-\frac{1}{3}}$

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Angular velocity ω and angular momentum L	$v = r\omega; \quad \omega = 2\pi f = 2\pi/T;$ $L = I\omega = mr^2\omega \quad$ (for orbiting bodies)
Centripetal acceleration and force	$a_c = \omega^2 r = \frac{v^2}{r}; \quad F_c = ma_c$
Kepler's 3rd law	$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$
Hydrostatic equilibrium	$\frac{\mathrm{d}P}{\mathrm{d}R} = -\rho_r \frac{GM_r}{R^2}$
Quantisation of energy–momentum	$E = hf = \hbar\omega; \quad p = \frac{h}{\lambda} = \hbar k$
Planck's law for intensity per unit frequency	$I_f = \frac{2\pi h f^3}{c^2} \frac{1}{e^{\frac{hf}{kT}} - 1}$
Stefan–Boltzmann law	$L = 4\pi R^2 \sigma T^4$
Wien's displacement law	$\lambda_{\max} = \frac{b}{T}$
Jeans length	$R_J = \sqrt{\frac{15 k_B T}{4\pi G \langle m \rangle \cdot \langle \rho \rangle}}$

Table 5: Relativistic Expressions

Description	Formula
Lorentz factor	$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$
Velocity addition	$u' = \frac{u+v}{1+\frac{uv}{c^2}}$
Time dilation and length contraction	$\Delta t' = \gamma \Delta t$ and $L' = \frac{L}{\gamma}$
Relativistic Doppler effect	$f_{\text{observed}} = f_{\text{source}} \cdot \sqrt{\frac{c-v}{c+v}}$
Relativistic redshift	$z = \sqrt{\frac{c+v}{c-v}} - 1 \approx \frac{v}{c}$
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$
Redshift	$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$

Table 6: Practical Astronomy

Description	Formula
Keplerian orbital ellipse as a function of angular deviation from periapsis	$r = \frac{a(1-\epsilon^2)}{1+\epsilon\cos\phi}$
Orbital eccentricity in terms of other parameters	$\epsilon = \frac{a - r_{\text{periapsis}}}{a} = \frac{r_{\text{apoapsis}} - a}{a} = \frac{r_{\text{a}} - r_{\text{p}}}{r_{\text{a}} + r_{\text{p}}}$
Rayleigh resolution criterion with aperture diameter D	$\sin\Delta\phi_{\text{min}} = 1.220\frac{\lambda}{D}$
Beam divergence angle with initial beam width D	$\delta = \frac{4\lambda}{\pi D}$
Rocket equation	$\Delta v = v_{\text{ex}} \log_e \frac{m_i}{m_f}$

Table 7: Distance Determination and Some Empirical Results

Description	Formula
Absolute bolometric magnitude	$M_{\text{bol}} = -2.5 \log_{10} \frac{L}{L_\odot} + 4.7554$
Distance modulus: difference between apparent and absolute magnitude	$m - M = 5 \log_{10} \frac{d}{10\ \text{pc}}$
Relationship between luminosity and absolute magnitude	$\frac{L_1}{L_2} = 10^{(M_2 - M_1)/2.5}$
Determining distance d in parsecs using an observed parallax p in arcseconds	$d \approx \frac{1}{p}$
Period–luminosity relationship for Cepheid variable stars, with period P in days	$M = -2.76 \log_{10} P - 1.4$
Absolute magnitude of RR Lyrae stars	$M \sim 0.75$
Absolute magnitude of Type Ia supernovae (at peak)	$M \sim -19.3$
Tully–Fisher relation	$L \propto V^4$
Mass–luminosity relation for main sequence stars	$L \propto M^{3.5}$
Hubble's law	$v = H_0 d$