AstroChallenge 2024
Senior MCQ Round

SOLUTIONS

Monday 3rd June 2024

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of a total of **38** printed pages, including this cover page.
2. Any materials other than the Question Paper and Formula Booklet are strictly prohibited.
3. Do **NOT** turn over this page until instructed to do so.
4. You have **2 hours** to attempt **ALL** questions in this paper. If you think there is more than one correct answer, choose the *most* correct answer.
5. At the end of the paper, submit the Optical Answer Sheet. You do not need to submit this booklet.
6. Please ensure that your name, school, and team number are clearly indicated in the Optical Answer Sheet.
7. It is *your* responsibility to ensure that your Optical Answer Sheet has been submitted.

1 On 2nd November 2023, Astronauts conducting a spacewalk on the International Space Station (ISS) accidentally dropped a tool bag. Tables 1 and 2 contains some information about the bag and the ISS.

Mass	19.46 kg
Shape	Cuboid
Dimensions	Approximately 40.64 cm × 45.72 cm × 10.16 cm
Average Apparent Magnitude	+6.0

Table 1: Information about the lost tool bag.

Mass	419,725 kg
Altitude	408 km
Orbital Period	92.6 min
Length	109 m

Table 2: Information about the ISS.

The tool bag is above the horizon in the sky at where Annette is at 2 pm local time in Singapore. Assuming that the bag follows the same orbit as that of the ISS, which of the following telescopes, if any, allows Annette to observe the bag at that time?

Telescope	Focal Length	Aperture
I	300 mm	70.0 mm
II	1000 mm	127 mm
III	800 mm	80.0 mm
IV	1500 mm	150 mm

(A) IV only

(B) II and IV

(C) I and II

(D) I, II, III and IV

(E) **None of the telescopes**

Solution:

None of the telescopes. At 2 pm local time, the Sun is still considerably high up in the sky and outshines most objects within observable range. As such, none of the telescopes outlined in the options will be able to help Annette observe the tool bag.

2 Which of the following observational equipment is based on similar principles to a total solar eclipse?

(A) Spectrometer

(B) Hydrogen-alpha filter

(C) **Occulting disk**

(D) Zenith telescope

(E) Charge-coupled device

Solution:

An occulting disk is a disk that is placed centrally at the focal point, or eyepiece of a telescope to block out the view of a significantly bright object, allowing fainter objects nearby to be seen more easily. This is similar to that of a total solar eclipse, where the moon blocks out direct light from the sun due to positional alignment, allowing fainter objects such as the corona to be observed.

3 Earth-Moon-Earth communication is a technique used by radio enthusiasts to send radio signals to fellow enthusiasts on other sides of the world. This technique uses the Moon as an object for radio signals to bounce off from. Which of the following statements about radio signals transmitted using Earth-Moon-Earth communication is true? Assume that the Earth and Moon have circular orbits.

(A) Radio signals transmitted via this method do not experience any form of doppler shifts throughout from transmission to receiving.

(B) **Radio signals transmitted via this method are subjected to higher levels of noise when the Moon crosses the galactic plane.**

(C) The strength of radio signals transmitted via this method remains constant regardless of the location on the Moon it is transmitted to.

(D) Statements A, B and C are true.

(E) None of statements A, B or C are true.

Solution:

Answer: B

The galactic plane contains a higher density of stars compared to the rest of the sky. Such results in the galactic plane being a more significant source of noise for radio transmissions in the form of radio waves from stellar radiation compared to the rest of the sky.

Meanwhile, motion of the Earth as well as the moon along their orbits results in a relative motion between the Earth and the Moon, resulting in Doppler shift occurring. As such, A is false.

The surface of the moon is also not perfectly smooth, and has craters, mountains and valleys which result in different elevations. Hence, the distance travelled by radio signals and hence their strength (as a result in the inverse square law) is dependent on the location on the moon the signal is transmitted to (for example, the distance travelled by a signal transmitted to the peak of a mountain would be lower than that of a signal that is transmitted to a crater). As such, B is false.

4 Solar sails have been a method of propulsion that is of interest to the space exploration community for a long period of time. Fitted with reflective sails, this method of propulsion seeks to utilise radiation pressure, derived from photons from the Sun to gain momentum. When an incident ray of photons (each carrying momentum p_{photon}) reflects off solar sails, the solar sail gains a momentum of at most $2p_{\text{photon}}$

Given a perfectly reflective and flat solar sail, and a uniform incidence of photons, determine the maximum force experienced by a solar sail at Earth's position with an area of $2\,\text{m}^3$.

(A) 4.56 µN

(B) **18.2 µN**

(C) 9.12 µN

(D) 36.4 µN

(E) Not possible to determine

Solution:

As outlined in the question, the momentum and hence force experienced by a solar sail is fully derived from radiation pressure, which is a result of the reflection of photons off the sail. Given the conditions outlined in the question, the maximum momentum a solar sail of any area A would receive is equivalent to

$$p_{\text{sail}} = 2p_{\text{incident photon}} = 2\frac{E_{\text{incident}}}{c} = 2\frac{AI_{\text{incident}}}{c}.$$

The momentum of the sail is twice that of the incident photon as a result of the principle of conservation of momentum, as the incident photon hits the sail along the normal of the sail and is reflected. Based on this result, and the definition of luminosity (which is in essence, power), we hence can infer that, given the Sun has a luminosity of L,

$$F_{\text{sail}} = \frac{\mathrm{d}p_{\text{sail}}}{\mathrm{d}t} = \frac{2AB_{\text{incident}}}{c}, \quad \text{where } B_{\text{incident}} = \frac{L}{4\pi d^2} = \frac{\mathrm{d}I}{\mathrm{d}t}.$$

Substituting the values appropriately will yield option B.

5 **Please refer to the Insert for Figure I-1.**

Figure I-1 is an unedited picture of the night sky taken by Abbel using his smartphone, over a 30 second exposure. The camera has a field-of-view of 70 degrees.

Figure I-1 contains a distortion causing the image to contain what appears to be a star trail. Which of the

following is the most likely to explain why this distortion is observed in the image?

(A) Light of different wavelengths refract at different angles when it enters the lens of the camera.

(B) The rotation of the Earth results in the position of the stars to shift.

(C) **The phone was not set in a fixed position, allowing the camera to shift due to some disturbances nearby.**

(D) Movement of air in the upper atmosphere results in the path of the light from the stars to deviate as a result of refraction.

(E) The phone used 2 cameras and failed to compile them properly.

Solution:

Answer: C

As mentioned in the preamble of the question, the distortion in the picture manifests as a form of a star "trail", which is a form of motion blur. This occurs when the object that a camera is taking a picture of, or the camera itself has been moved when the shuttle is opened. Such eliminates the possibility of chromatic aberration (alluded by (a)) and twinkling of stars (alluded by (d)) to be the leading reason for the distortion shown.

6 On one fateful day, a star of known brightness was observed to have been occulted by an asteroid, causing the star's observed brightness to be reduced by half.

What is the change in the star's apparent and intrinsic absolute magnitudes before and during the occultation? You may take that the star's initial brightness with respect to Earth was measured in the absence of any interfering events or phenomena (such as occultations with other celestial bodies, planetary transits etc).

Option	Change in Apparent Magnitude	Change in Intrinsic Absolute Magnitude
(A)	0.7526	0
(B)	0.3763	0.7526
(C)	0	0.7526
(D)	0.1815	1
(E)	0	0

Solution:

Answer: A

By definition, magnitudes of objects are a logarithmic scale which describes the brightness of an object. An object of magnitude 1 is 100 times brighter than that of an object of magnitude 6. This means that $100 = x^5$.

Or, more generally, a difference in magnitude Δm between two objects implies that an object is $x^{\Delta m}$ times brighter than another object. With some mathematical manipulation, we see that $x = \sqrt[5]{100}$.

In the context of the question, we see that the star's brightness was reduced by half, we can see that the change in *apparent* magnitude Δm can be expressed as

$$2 = \left(\sqrt[5]{100}\right)^{\Delta m}$$

and solving for Δm gives us $\boxed{0.7526}$.

However, the story is a little different for figuring out the changes in the *intrinsic absolute magnitude*, which refers to the magnitude of an object as observed by an observer 10 parsecs away from the object. Since the intrinsic absolute magnitude of the star is calculated using its brightness with respect to Earth, and it was given in the question that the brightness of the star without any interfering events and phenomena is known, the intrinsic absolute magnitude of the star does not change.

7 Upon asking AstroChallenge's newly developed prototype AI chatbot about certain fun facts about variable stars, the AI chatbot produced the following three statements:

I The nearest variable star in the night sky is Proxima Centauri.

II Eclipsing binaries are considered to be a type of variable star.

III Only red dwarves, main sequence stars, giants and supergiants are capable of being variable stars.

Which of the following statements are true?

(A) **I** only

(B) **II only**

(C) **III** only

(D) **I** and **II**

(E) **I** and **III**

Solution:

Answer: B

By definition, a variable star is a star that experiences changes in brightness over time, be it due to internal or external factors. As the brightness of eclipsing binaries do change over time as a result of each member of the binary eclipsing each other, such binaries can indeed be considered as a type of variable star.

Proxima Centauri is, as its name suggests, the nearest star from Earth, and is known to be a flare star which experiences random increases in brightness through what is thought to be a stellar flare.

Since variable stars are stars that experience changes in brightness over time, not only the stars outlined in statement (iii) can be considered as variable stars—stars of all stages can be considered as variable stars.

8 In 1054, Chinese astronomers recorded the appearance of a "guest star" in the sky. This event is now

known to have been a supernova. Which of the following celestial objects was created by this supernova?

(A) Cat's Eye Nebula (NGC 6543)

(B) **Crab Nebula (M1)**

(C) Lagoon Nebula (M8)

(D) Eagle Nebula (M16)

(E) Ring Nebula (M57)

Solution:

Answer: B

The Crab Nebula was the first identified astronomical object to correspond with a historically observed supernova. Besides, it is the only supernova remnant among the objects listed.

9 It is known that the total energy E of an object bound in an elliptical orbit is given by

$$E = -\frac{GMm}{2a}.$$

Which of the following is the correct expression for the velocity of a satellite orbiting the Earth at a given point in time?

[Note: r = distance between satellite and center of Earth; a = semi-major axis of orbit]

(A) $v = 4GM_{\text{Earth}}\left(\frac{2}{r} - \frac{1}{a}\right)$

(B) $v = \sqrt{GM_{\text{Earth}}\left(\frac{2}{r} - \frac{1}{a}\right)}$

(C) $v = \sqrt{GM_{\text{Earth}}\left(\frac{1}{a} - \frac{2}{r}\right)}$

(D) $v = \sqrt{GM_{\text{Earth}}\left(\frac{1}{r} - \frac{1}{a}\right)}$

(E) $v = \sqrt{GM_{\text{Earth}}\left(\frac{2}{r} + \frac{1}{a}\right)}$

Solution:

Answer: B

Potential energy of the satellite is given by $U = -GMm/r$. By conservation of energy, we have $K = E - U = \frac{1}{2}mv^2$. Making v the subject gives us $v = \sqrt{GM_{\text{Earth}}\left(\frac{2}{r} - \frac{1}{a}\right)}$. This is precisely the *vis–viva equation*.

10 Suppose a satellite is orbiting around a planet of radius 6000 km. The satellite reaches periapsis at 500 km above the planet's surface while travelling at a speed of 10 km/s. Determine the speed of the

satellite when it reaches apoapsis, at an altitude of 7000 km.

(A) 0.7 km/s

(B) 10 km/s

(C) **5.0 km/s**

(D) 7.0 km/s

(E) 15 km/s

Solution:

The angular momentum L_p of the planet–satellite system at periapsis is given by $L_p = mv_p r_p$. Likewise, the angular momentum of the system at apoapsis is given by $L_a = mv_a r_a$.

By the conservation of angular momentum, we must have $L_p = L_a$. Solving for v_a will give us $\boxed{5.0\,\text{km/s}}$.

11 A rotating black hole with no charge is observed to have a mass of about 36 solar masses. Which of the following statements about the black hole is correct?

I If the black hole was non-rotating, it would have a Schwarzschild radius of approximately 106 km.

II For rotating black holes, if a singularity exists, general relativity predicts the singularity to have a ring–like shape.

III If it was charged, it would be reclassified as a Kerr–Newman black hole instead.

IV The black hole was formed when a star the size of our Sun went supernova.

V Since the black hole is so massive it should be called a supermassive black hole.

(A) **I** and **IV**

(B) **III** and **V**

(C) **I**, **III** and **IV**

(D) **I, II and III**

(E) All of the above statements are correct.

Solution:

Answer: D

Let us evaluate each statement:

I The Schwarzschild radius of a black hole is given by $r = 2GM/c$. Given that $M = 36M_{\text{Sun}}$, solving for r would give us approximately 106 km.

II Factual recall.

III Factual recall.

IV The mass of the Sun is insufficient to allow for the formation of a black hole after its death. For this, recall that the Chandrasekhar limit, the mass limit in which a star will turn into a neutron star or black hole, is about $1.4M_{\text{Sun}}$. The death of our Sun would actually result in a white dwarf.

V Supermassive black holes need to have masses above 10^5 solar masses.

12 It is known that a galaxy, AC 2024, has an active galactic nucleus. It was observed that the peak wavelength of the OIII primary transition line of 500.7 nm has been redshifted to 523.1 nm. How far away does AC 2024 lie from us? (Assume that the Hubble's Parameter is given as $H_0 = 67.8\,\text{km/s/Mpc}$)

(A) 72 Mpc

(B) 128 Mpc

(C) **198 Mpc**

(D) 146 Mpc

(E) 102 Mpc

Solution:

Answer: C

Let z be the redshift. We have $\lambda_{\text{observed}} = (z+1)\lambda_{\text{emitted}}$. Solving for z gives us $z = 0.04473$. Since z is small, $v/c \ll 1$. This allows us to use the approximation $z = v/c = H_0 d$, which gives us $d = \boxed{198\,\text{Mpc}}$.

13 One potential reason that intelligent life is rare or even a one-off event, is that there exist factors that prevent the development of such life or prematurely ends them. This is known as the *Great Filter Hypothesis*. The concept originates from Robin Hanson. He proposes that there nine steps in the evolutionary path. Here are six of the nine steps outlined:

1 Cell-based life

2 A civilization advancing toward the potential for a colonisation explosion

3 The right star system (potentially habitable planets)

4 Colonisation explosion

5 Tool-using animals with intelligence

6 Organic molecules

Arrange these steps in the correct order to form Robin Hanson's "evolutionary path".

(A) **3, 6, 1, 5, 2, 4**

(B) **3, 1, 4, 6, 5, 2**

(C) **3, 1, 6, 4, 5, 2**

(D) **1, 2, 3, 4, 5, 6**

(E) **5, 3, 6, 2, 1, 4**

Solution:

Answer: A

14 **Please refer to the Insert for Figure I-2.**

What is a plausible reason that explains why the stars appear as small circles instead of a point?

(A) **The image is out of focus**

(B) Gravitational lensing

(C) Light pollution

(D) Rotation of the Earth

(E) Those are Solar System objects, not stars

Solution:

(A) **is correct.**

(B) is not relevant.

(C) would result in less star being observed and less contrast between the background sky and the star. It would not result in stars looking like circles.

(D) would result in star trails as stars would apparently rotate around the celestial poles.

(E) is not relevant, as both Solar System objects and stars are both able to suffer from this.

15 Halley's Comet orbits the Solar System once every 75 to 79 years even though it undergoes a rather well-defined orbit, which of the following is/are the reason(s) why the orbit does not have a constant period like Earth?

I The influence of gas giants like Jupiter

II The perturbation of trans-Neptunian objects

III Outgassing as it approaches the Sun

IV Breaking away of small pieces along Earth's orbital path

V The high eccentricity of the orbit causing precession

(A) **I** only

(B) **I, II, IV**

(C) **I, III, V**

(D) **I, II, III**

(E) **I, II, III, IV**

Solution:

Answer: E

16 In 1783, John Michell hypothesised the existence of dark stars, a type of star that has gravity so strong that light emitted by the star would travel a small distance before falling back onto the surface, rendering it invisible to observers. At that point of time, Michell believed that light acted solely as particles, a theory that was popular during his and Newton's time.

Upon learning about the concept of dark stars, Alson came to the following conclusions:

I A dark star of the same mass as the Earth would have a maximum radius of about 0.886 cm.

II Photons emitted by the dark star would experience a loss in velocity as a result of the gravity of the star, causing the photons to experience redshift.

III Despite light not being able to escape from the dark star, it is still possible to determine the presence of a dark star, even in the absence of another star near the dark star.

Using our current understanding of astronomy, which of the statements made by Alson is/are **FALSE**?

(A) **I** only

(B) **II only**

(C) **III** only

(D) **I** and **III**

(E) None of the statements are false

Solution:

While the idea of a dark star may seem rather unfamiliar, the idea that light is unable to escape from the star means that the concept of dark stars is similar to that of a black hole—in fact, if we were to take into account of our current understanding of Astronomy and apply it to the case of a dark star, it is virtually indistinguishable from that of a black hole!

Since light is unable to escape from a dark star, the escape velocity of the star from its surface is equal or greater than the speed of light. As such, the maximum radius of a dark star of the mass of the Earth is

$$v_{\text{escape}} \geq c \geq \sqrt{\frac{2GM}{r}}, \quad r \leq \frac{2GM}{c^2}.$$

Substituting the values accordingly would hence show us that the maximum radius of a dark star

with the mass of the Earth is about 0.00886 m. As such, statement **I** is indeed true.

As light travels away from a region of high gravity, like the region around a dark star, it loses energy which results in a corresponding change in the frequency of light (as outlined by the Planck–Einstein Relation, $E = hf$). However, the speed of light is always constant under all conditions and hence, the redshift does not derive from the light's velocity decreasing. As such, statement **II** is false.

Objects other than stars that radiate light, such as the presence of an accretion disc and relativistic jets from a dark star, as well as its gravitational effects (leading to phenomena like Gravitational Lensing) due to the star's immense gravity can also allow us to infer the presence of a dark star even without the assistance of other stars near the dark star. As such, statement **III** is true.

17 What is the correct explanation of the statement "stars rise approximately four minutes earlier every day"?

(A) One sidereal day is approximately four minutes longer than one mean solar day.

(B) **the Earth orbits around the Sun, hence the relative positions of the Sun, the Earth and distant stars change every day.**

(C) Earth's rotation results in observers looking at different areas of the sky at different times.

(D) The Moon slows the rate of Earth's rotation down by approximately four minutes every day.

(E) This statement is not true. Stars do not rise during the day and hence it does not make sense to say that stars "rise earlier every day" as eventually its "rising time" would be before Sunset.

Solution:

(A) is wrong as one sidereal day is approximately 4 minutes shorter than one mean solar day. Had it been "shorter" than "longer", this would have been a correct explanation.

(B) is correct. This is why one sidereal day is shorter than one mean solar day.

(C) explains why stars rise and set. It does not explain the difference between one sidereal day and solar day (difference in time between stars and the sun rise and set)

(D) is wrong. Although the moon does slow down the rate of rotation of the Earth, it is not this significant. Also, it is irrelevant in this discussion.

(E) is wrong. It does make sense for stars to rise at a time that the sun is above the horizon. It would be in the sky, but the sun would be so bright that we would not be able to observe the star.

18 When Jerry asked Tom where Aldebaran was, Tom took out his compass and pointed it in the direction of Aldebaran. When comparing the position of Aldebaran to the compass, he found that Aldebaran was at 165 degrees. This implies that the _____ of Aldebaran is 165 degrees.

What should fill in the blank?

(A) Longitude

(B) Latitude

(C) **Azimuth**

(D) Altitude

(E) Declination

Solution:

Answer: C

In the altitude–azimuth coordinate system, azimuth is the angular displacement of a celestial object from cardinal north. As both the altitude-azimuth and the compass has the observer as the centre and the horizon as the reference plane, azimuth and the numbers (which represent the bearing) on the compass is equivalent.

19 Why are radio telescopes normally so much bigger than visual ones?

(A) For the wavelengths most radio telescopes observe at, a large majority of the waves would be absorbed by the atmosphere. Hence a larger aperture is required.

(B) Building larger radio telescopes means that it is much easier for aliens to notice us. In comparison, a large visual telescope would not afford such an opportunity for aliens.

(C) A large radio telescope would pick up less radio noise generated from our communication devices.

(D) **Longer wavelengths of radio waves and the diffraction of radio waves dictates that a larger aperture is required to achieve the same resolution.**

(E) None of the above. Large radio telescopes just look cooler than visual telescopes so people build more of them.

Solution:

(A) is wrong as radio telescopes are built on Earth and hence would be observing at frequencies that are not observed by the atmosphere.

(B) is wrong and irrelevant in the discussion.

(C) is wrong. To reduce the radio noise generated by our communication devices, radio telescopes are built in remote areas. Size of the telescope does not affect the amount of noise.

(D) is correct. Rayleigh criterion states that the angular resolution is inversely proportional to the wavelength of the incoming signal and directly proportional to the aperture of the telescope. Hence, with a longer wavelength compared to visible light, it would require a larger aperture to achieve the same angular resolution.

(E) is wrong.

20 The following profile was obtained from sampling the 21-cm emission line of neutral hydrogen the entire spiral galaxy NGC 3198 at the same instant. We can assume the galaxy is observed almost edge-on. Here, the heliocentric velocity refers to the radial velocity of the galaxy with respect to the Sun.

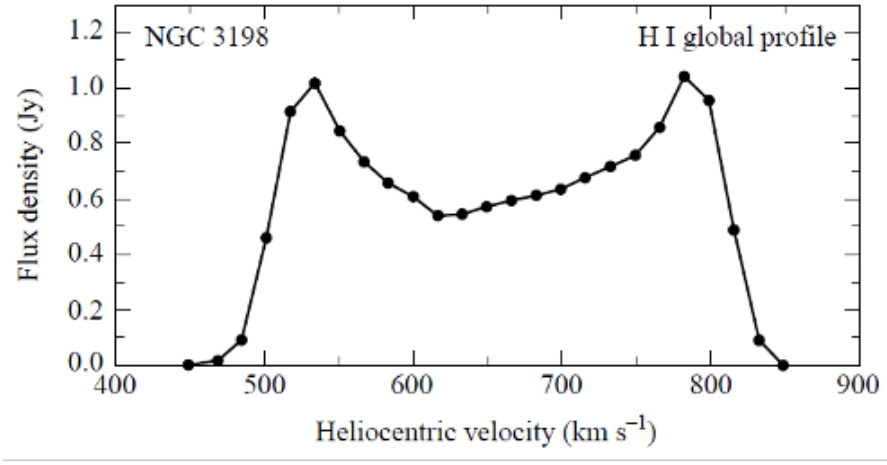

Figure 3: 21-cm emission line profile of NGC 3198.

Which of the following reasons correctly explains the existence of a double peak in the profile?

(A) There is another galaxy in the foreground of NGC 3198 with a lower Heliocentric velocity. Thus the observed 21-cm emission is of both galaxies superposed together, resulting in the double peak.

(B) The double peak is a result of the transit of another object in front of NGC 3198 during the sampling period. During the sampling of the region 525–875 km/s, a celestial object transited NGC 3198, reducing the intensity of 21-cm radiation reaching the detector, and thus manifesting as a drop after the first peak. In reality, this profile should only have a single peak.

(C) There is a peak due to the flat rotation curve of the galaxy. Most of the neutral hydrogen is rotating at that flat-rotation velocity and thus, the flux density at that velocity is the highest. The peak is double because the first corresponds to the center of galaxy, whereas the second corresponds to the edge of the galaxy.

(D) There is a peak due to the flat rotation curve of the galaxy. Most of the neutral hydrogen is rotating at that flat-rotation velocity and thus, the flux density at that velocity is the highest. As one half of the galaxy is rotating away from us and the other towards us, the 21-cm line is red shifted and blue shifted respectively resulting in two peaks.

(E) This is because of random fluctuations due to virtual particles in the interstellar medium between NGC 3198 and us.

Solution:

(A) is not possible because if there was a galaxy in the foreground, it would have been extremely difficult to detect NGC 3198 in the first place.

(B) is nonsensical because as the question states, the profile was taken at one instant of a time. Thus, transit is impossible.

(C) is wrong because while the explanation for the peak is correct, the explanation for the double peak is nonsensical. Both peaks represent the flat rotation curve of the galaxy.

(D) is correct as mentioned.

(E) is just nonsense.

21 An astronomer at the north pole is interested in viewing the same star continuously for 24 hours. His setup includes an optical tube assembly (with focal length 650 mm) and an eyepiece (focal length 20 mm and AFOV 65°). If he aligns his setup such that Polaris is located at the center of the eyepiece, stars with what declination will he be able to observe for 24 hours?

(A) **Declinations greater than +89°.**

(B) Declinations less than −89°.

(C) Declinations greater than +65°.

(D) Declinations greater than +78°.

(E) It is impossible to see a star for 24 hours straight.

Solution:

Answer: A

First we need to calculate the true field of view of his setup

$$\text{TFOV} = \frac{\text{AFOV}}{M} = \frac{\text{AFOV} \times f_{\text{eyepiece}}}{f_{\text{OTA}}} = \frac{65 \times 20}{650} = 2°.$$

Given the TFOV is so small, we can assume the spherical surface of the celestial sphere to be approximately flat in the eyepiece. Therefore, he can only see stars with a declination greater than +89°, as Polaris with declination approximately 90° is located at the center of the eyepiece.

22 Which of the following statement is true about core-collapse supernovae?

(A) After the formation of the iron core, the photons gain enough energy to cause photodisintegration of the iron back into protons and neutrons, which is a highly exothermic process. The extremely huge amount of energy released at the core results in a shockwave extending outwards from the core, resulting in the supernova.

(B) After the formation of the iron core, the core further causes fusion of iron into cobalt through the r-process assisted by neutrinos. Beyond this, cobalt can no longer fuse into any heavier nuclei, hence resulting in the collapse of the core, leading to the supernova.

(C) After the formation of the iron core, the first, second and third dredge-up processes causes iron to be removed from the core into convection zones. This reduces pressure in the core and hence causes the core collapse resulting in a supernova.

(D) After the formation of the iron core, the star becomes unstable and enters the instability strip, leading to boomification and the start of core collapse.

(E) If the iron core exceeds the Chandrasekhar Limit, the photons gain enough energy due to extreme core temperatures to start photodisintegration of iron which is highly endothermic. This decreases the thermal energy in the core required to support the core. Thus, starting a core collapse and eventually leading to a supernova.

Solution:

(A) is wrong because photodisintegration is endothermic.

(B) is wrong because in terms of binding energy, the core does not fuse beyond iron.

(C) is wrong because the dredge up processes takes place when the star leaves the Main Sequence branch and becomes highly convective, resulting in transport of elements from the inner core to the outer layers.

(D) is nonsense.

(E) is correct.

23 What is the approximate difference in right ascension between the Moon and the Sun during a full moon?

(A) 0 h

(B) 8 h

(C) 12 h

(D) 16 h

(E) 24 h

Solution:

Answer: C

24 In solar astrophotography, interference filters called etalons are used to look at specific layers of the Sun because each layer has a different peak emission wavelength. For example, the Sun's photosphere emits mainly in the hydrogen-alpha (H-α) range.

To observe the Sun's photosphere, a solar astrophotographer used two H-α etalons stacked on top of each other. If each etalon has a bandwidth of 0.7 Å, the combined bandwidth would be 0.49 Å.

When asked the reason for using two etalons, he replies "it decreases the bandwidth and thus we can prevent noise from wavelengths other than H-α." However, he says he will not use a triply-stacked etalon. What is the reasoning behind his decision?

(A) A triply-stacked etalon will cause the setup to become too heavy on one side, and may tilt the setup during imaging.

(B) A triply-stacked etalon will not reduce the bandwidth any further, and is a waste of money.

(C) As etalons work on interference, using three will cause destructive interference and thus nothing will be observed.

(D) A triply-stacked etalon will result in the peak emission wavelength of the Sun to switch to the Calcium-K region, and thus will help us observe the Sun's chromosphere instead of the Sun's photosphere.

(E) A triply-stacked etalon will decrease the intensity of light reaching the sensor, thus reducing the brightness and details of features in the image.

Solution:

(A) is wrong because if your telescope setup is unbalanced, you can shift the telescope and balance it again. Hence, this is a rather nonsensical answer.

(B) is wrong. The bandwidth will be reduced.

(C) is wrong because the interference is taking place inside the etalon and not across etalons.

(D) is nonsense.

(E) is correct as stated.

25 An observer is located at 21°N latitude at night, when Crab Nebula (M1) is at its upper culmination. Which of the following set of objects from Table 3 can he observe? Assume that he is using a telescope with an aperture of 100 mm, and the limiting magnitude of this telescope can be approximated as

$$2.2 + 5\log_{10}(A)$$

where A is the telescope aperture in cm.

Object	Right Ascension	Declination	Apparent Magnitude	Absolute Magnitude
47 Tuc	0 h 24 m 5 s	−72° 5′ 13.0″	4.09	−9.42
M36	5 h 36 m 19 s	+34° 9′ 11.6″	6.00	−4.62
M1	5 h 34 m 33 s	+22° 0′ 53.1″	8.40	−3.1
M65	11 h 18 m 56 s	+13° 5′ 27.3″	10.25	−18.78
NGC 292	0 h 52 m 39 s	−72° 48′ 20.5″	2.20	−17.11
NGC 1909	5 h 2 m 1 s	−7° 54′ 0.8″	8.00	−7.92

Table 3: Question **25**.

(A) M1 only

(B) **M36 only**

(C) M1 and M36

(D) NGC 1909, M1, 47 Tuc, and M36

(E) 47 Tuc and NGC 292

Solution:

Answer: B

First we calculate the limiting magnitude

$$L_M = 2.2 + 5\lg(10) = 7.2.$$

Thus, M1, M65 and NGC 1909 are eliminated. Then we can look at the declination limits. As the observer is located at a latitude of 21°N, anything with a declination of less than −69° cannot be observed. Thus 47 Tuc and NGC 292 are eliminated. Hence, he can only observe M36.

26 When observing the Sun at optical wavelengths, the edges of the solar disk appear dimmer relative to the centre. This phenomenon is called limb darkening. Which of the following best explains this

phenomenon?

(A) The photosphere of the Sun is optically thin (translucent). Therefore, we can only see the thin photosphere when viewing the edge of the Sun, and not the thicker radiative zone. Darkening occurs because we are looking at space through the edge of the Sun.

(B) **Our line of sight intersects the edge of the Sun at a shallower angle. Thus, the maximum optical depth along this line of sight lies at a shallower radial depth, where the temperature of the Sun is lower. By the Stefan–Boltzmann law, the total intensity of light is lower.**

(C) The centre of the Sun is closer to Earth relative to its edges due to its spherical shape. By the inverse square law, the intensity of the edge of the solar disk falls off faster than the centre.

(D) The surface of the Sun near its edge is angled at a shallow angle relative to Earth. Therefore, the intensity of light decreases by a factor of $\cos\theta$, where θ is the angle between our line of sight and the tangent plane of the surface.

(E) The limb darkening effect is actually caused by an optical effect in telescopes called vignetting, where off-axis regions in the image plane appear darker than the centre due to the lower photo flux per unit area.

Solution:

(A) is wrong because the entire disk of the sun (photosphere) is optically thick throughout. In fact, the photosphere is defined by the (average) layer at which the sun's atmosphere is optically thick.

(B) is correct.

(C) is wrong because the differences in distances are negligible relative to the Earth-Sun distance.

(D) is wrong because the photosphere of the sun emits light in all directions, and not in coherent parallel rays.

(E) is wrong for obvious reasons.

27 Which of the following statements is/are true?

I The rotational axis of the Earth precesses in a circle centred about the north ecliptic pole.

II To an observer on Earth, the north celestial pole appears to precess in a counterclockwise direction.

III As a result of precession, the position of the vernal equinox appears to shift eastward over time.

(A) **I** only

(B) **III** only

(C) **I** and **III**

(D) **II** and **III**

(E) **I and II**

Solution:

I is true; precession is caused by torque exerted by the sun and moon, which lie on the ecliptic plane.

II is true; this can be determined by using the right-hand rule to determine the direction of the torque exerted.

III is false; on the celestial sphere, counterclockwise motion is associated with a westwards drift. For instance, stars rise from east to west (and thus travel westwards), and thus they appear to rotate counterclockwise about the North celestial pole.

28 One of the leading candidate theories on the formation of the Moon argues that the Moon was formed when an early protoplanet collided with Earth, and the Moon coalesced from the debris that was flung out during the collision. This theory was named ...

(A) The Great Heavy Bombardment

(B) **The Giant Impact Hypothesis**

(C) The Genshin Impact Hypothesis

(D) The Nice Model

(E) The Fission Theory

Solution:

Answer: B

Factual recall.

29 The Kirkwood gaps are narrow gaps in the asteroid belt that are nearly devoid of asteroids. Figure 4 is a plot of the number of asteroids against orbital radius around the Sun illustrating this phenomenon.

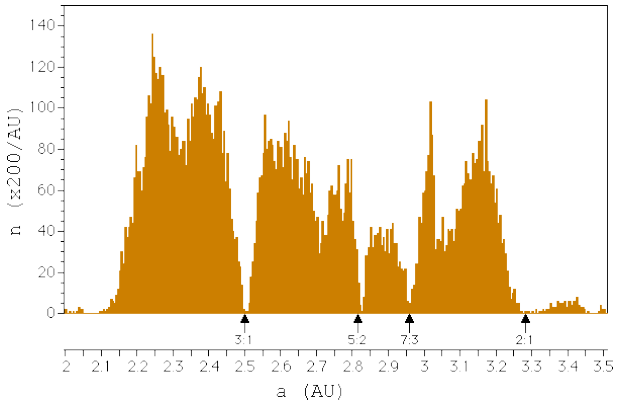

Figure 4: Question **29**.

Which of the following best explains the occurrence of these gaps?

(A) These gaps are carved out by the gravitational influence of minor planets in the asteroid belt, such as Ceres, Vesta, and Pallas, which act as 'shepherds'.

(B) These gaps are remnants of early Solar System planet formation, where the planetesimal cores of Mars and Jupiter formed before migrating to different orbits.

(C) The orbits of bodies at these gaps are in orbital resonance with Jupiter, and few stable orbits exist in these regions.

(D) These gaps occur at the Lagrange points of the Sun–Jupiter system. Objects in these gaps tend to become trapped in these Lagrange points, thus clearing these regions of asteroids.

(E) The Kirkwood gaps are created by a strong cleave from a famous US politician.

Solution:

(A) is wrong since the orbits of the minor planets are not associated with the Kirkwood gaps.

(B) is wrong because even if such gaps existed in the early solar system, they would have been filled by subsequent migrations.

(C) is correct.

(D) is wrong because the asteroid belt is nowhere near Jupiter's Lagrange points.

(E) is nonsense.

30 *"On the morning of 20th April, we can see Venus rise to its highest point in the sky before reversing its course. Over the next few months, Venus will drift closer and closer to the Sun, until it sets below the horizon and passes* **A** *the Sun, reappearing in the evening sky. After a period of roughly* **B** *, Venus will again reach its highest point in the evening sky."*

What should replace **A** and **B** respectively?

(A) in front of; 2 months

(B) behind; 2 months

(C) in front of, 5 months

(D) behind; 5 months

(E) Not enough information in the passage

Solution:

Answer: D

Remember that all planets orbit the Sun in a counterclockwise direction when viewed from the North pole. Earth also rotates on its own axis in a counterclockwise direction. Morning occurs when a point on the Earth's surface rotates towards the Sun. Therefore, for Venus to be visible in the morning sky, it must be 'to the right' of the Sun when viewed from Earth. Therefore, Venus is moving

towards the far side of its orbit, and will eventually pass behind the Sun.

Since maximum elongation occurs at points in the orbit that are closer to inferior conjunction, it will take a longer time for Venus to make the round trip around the Sun and reach maximum elongation again in the evening.

31 In the Morgan-Keenan classification for stars, stars are assigned a luminosity class in the form of roman numerals ranging from I to V. Luminosity class I stars are luminous supergiants, and the luminosity associated with each class decreases until V, which represent main-sequence stars. Astronomers can classify stars into these luminosity class purely based on their spectrum. Which features of stellar spectra do astronomers use to classify stars by their luminosity class, and why?

(A) Classification is done according to the width of absorption lines in stellar spectra. This is because the pressure of the stellar atmosphere of main sequence stars are lower than that of giant stars, resulting in pressure broadening of spectral lines.

(B) **Classification is done according to the width of absorption lines in stellar spectra. This is because the pressure of the stellar atmosphere of main sequence stars are higher than that of giant stars, resulting in pressure broadening of spectral lines.**

(C) Classification is done according to the abundance of heavy metal lines in the stellar spectra. This is because giant stars are late-stage stars, and they have abundant metals in their atmosphere as a result of the fusion of elements heavier than hydrogen in the core.

(D) Classification is done according to the abundance of heavy metal lines in the stellar spectra. This is because giant stars have cooler outer atmospheres, and therefore metals are not thermally ionized and can produce absorption line features instead of continuum features.

(E) Classification is done according to the presence of hydrogen-alpha emission lines in the stellar spectra. This is because giant stars have largely shed their stellar atmospheres, which in turn are ionised by the stellar wind, resulting in hydrogen alpha emission.

Solution:

Answer: B

The surface gravity of late-stage stars is lower as its atmosphere is less gravitationally bound to the star, resulting in a smaller pressure gradient when in hydrostatic equilibrium.

32 The rotational axis of the Earth precesses with a period of about 26,000 years. Which of the following

best explains why this precession occurs?

(A) **The equatorial bulge of the Earth is tilted with respect to the plane of the Solar System. Therefore, the gravitational pull of the Sun and the Moon exerts a torque in a direction perpendicular to the rotational axis of the Earth, causing precession to occur.**

(B) The rotation of the Earth has slowed since its formation. As the rotation slows, angular momentum is transferred to the precession of the Earth's rotational axis, similar to how tops precess as they 'spin down'.

(C) The equatorial bulge of the Earth is tilted with respect to the plane of the Solar System. Therefore, the gravitational pull of Jupiter exerts a torque in a direction perpendicular to the rotational axis of the Earth, causing precession to occur.

(D) An object in rotational motion is naturally unstable, and left on its own, its rotational axis tends to precess regardless of external perturbations.

(E) Precession occurs due to spin-orbit resonance between the Earth and the Moon.

Solution:

(A) is correct.

(B) is wrong, the precession of tops is caused by the moment exerted by the surface normal force and gravity rather than some 'transfer' of angular momentum.

(C) is an insufficient explanation as the gravitational influence of the Jupiter is far smaller than the sun and moon.

(D) is wrong; due to rotational inertia, an object in rotational motion has a stable axis of rotation.

(E) is wrong because there is no spin-orbit resonance between the Earth and the moon.

33 In long-slit spectroscopy, a slit aperture mask is situated over the central axis of an object such as a galaxy. Light from the slit is later split into its component spectrum in the axis perpendicular to the slit.

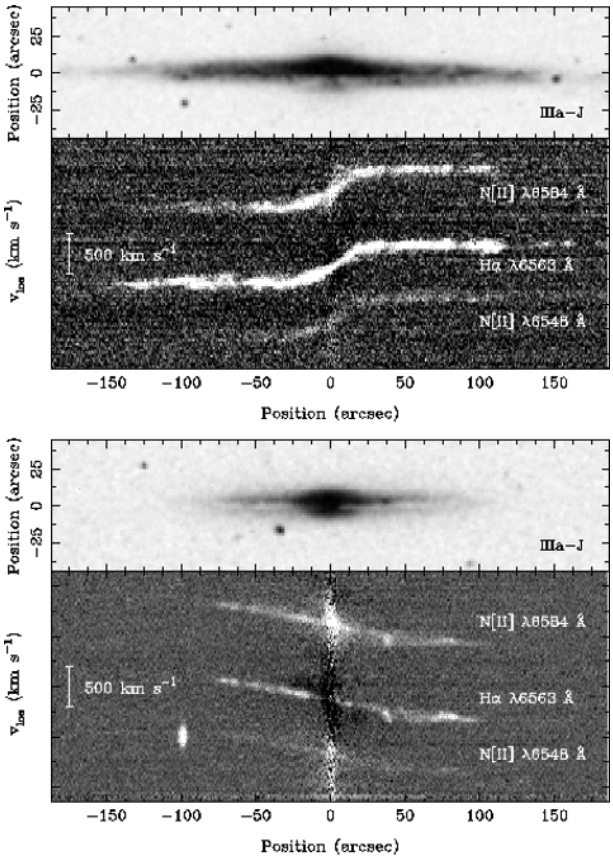

Figure 5: Question **33**.

Figure 5 is an example of the long-slit spectroscopy technique applied to a spiral galaxy. The black-on-white part of the diagram is an image of the original galaxy being imaged, while the white-on-black part of the diagram shows the emission lines at the wavelengths of 6584 Å, 6563 Å and 6548 Å, corresponding to N-II, H-alpha, and N-II emission lines respectively.

What physical property of the spiral galaxy can we deduce from its long-slit spectrum?

(A) **The rotation curve of the galaxy**

(B) The luminosity of the galaxy

(C) The average gas temperature of hydrogen clouds in the galactic arms

(D) The metallicity of the galaxy

(E) The angular inclination of the galaxy

Solution:

Answer: A

Long-slit spectroscopy is essentially a compact way to perform doppler spectroscopy sampled at multiple points along the main axis of the galaxy, which is rotating towards or away from the observer with rotational velocity depending on its radial distance from the centre of the galaxy. The overall rotation curve can be deduced if the inclination of the galaxy relative to Earth is known.

34 Figure 6 summarises the results of observations of the Bullet Cluster, which consists of two colliding clusters of galaxies. In the Figure, the green contour lines represent the mass distribution of the clusters as determined by gravitational lensing measurements, with the highest concentration of mass towards the central red concentric contours. The blue-red heatmap in the background represents the density of hot gas and plasma associated with the galaxy clusters, as measured by Chandra x-ray observations.

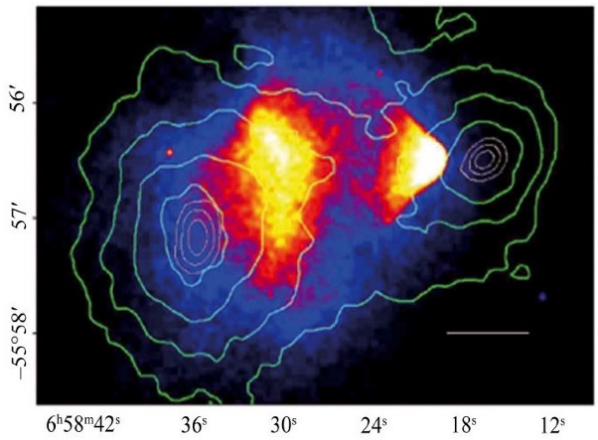

Figure 6: Question **34**.

These observations of the Bullet Cluster are often cited as a "smoking gun" for the correctness of dark matter over modified Newtonian dynamics (MOND) theories. Which of the following **best** explains why?

(A) In galaxy collisions without dark matter, we expect the hot gas and plasma to be "quenched" during the collision and cease to emit intense radiation. Therefore, the presence of strong x-ray radiation is evidence for the presence of dark matter.

(B) The diagram shows very little ram-pressure stripping of gas and plasma in the galactic collision than expected if only the gravity of visible baryonic matter is considered. This suggests that there is additional invisible mass holding onto the gas and preventing it from being stripped away.

(C) The contours show the presence of gravitational influence that is not coupled to light emitted by baryonic matter (hot gas and plasma). MOND cannot explain such gravitational influences in the absence of visible baryonic matter.

(D) Baryonic matter interacts strongly with one another, while dark matter is weakly interacting. Therefore, in collisions, we expect gas and plasma to be slowed down in the collision significantly while dark matter simply passes through each other. MOND cannot explain why baryons and dark matter interact differently in collisions.

(E) Nope, the Bullet Cluster does not actually pose any difficulties for MOND theories at all. No, I am not Mordehai Milgrom. Why are you asking this?

Solution:

(A) is pure nonsense.

(B) is not a sufficient explanation because it simply proposes that there is additional gravitational influence that is unaccounted for, which is the original form of the dark matter hypothesis/problem.

(C) is insufficient for the same reason.

(D) is correct.

(E) is nonsense.

35 If the Sun vanishes right now, the Earth will start moving in a path tangential to its orbit instantaneously according to Newtonian gravitation. However, general relativity predicts this will not be instantaneous and a time delay will exist. Keeping in mind that general relativity states nothing can travel faster than the speed of light, find the angular separation θ of the path taken by the Earth as predicted by Newtonian gravitation and general relativity (refer to Figure 7). You may assume the orbit of the Earth around the Sun is perfectly circular, and gravitational effects of other planets are negligible.

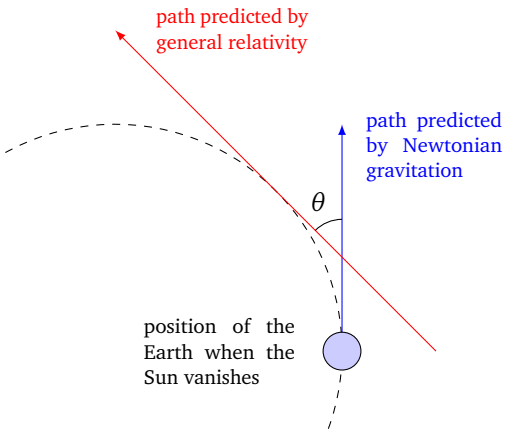

Figure 7: Question **35**.

(A) 0°

(B) 0.005,69°

(C) 0.0728°

(D) 0.100°

(E) Insufficient information to determine the angular separation.

Solution:

Answer: B

First, we need to calculate the time taken for the Earth to experience the effects of the Sun's vanishing. This is the same time it takes sunlight to reach earth, as information will travel at the speed of light

as well.

$$t = \frac{1.496 \times 10^{11}}{c} \approx 499.01\,\text{s}.$$

In this time, the Earth would have moved on its orbit by an angle α. It can be proved by geometric construction that $\alpha = \theta$. Thus,

$$\theta = \alpha = \frac{360^\circ}{365.24 \times 24 \times 60 \times 60} \times 499.01 \approx \boxed{0.00569^\circ}.$$

36 In Figure 8, the size of the circle represents the apparent magnitude of the star as viewed from the Earth. Which of the following is **NOT** a star?

Figure 8: Question **36**.

Solution:

Answer: A

(A) is not a star as there is no star with an apparent magnitude of 0 like Vega, Altair, or Deneb near Delphinus.

(B) is Altair.

(C) is Sadr.

(D) is Deneb

(E) is Vega.

Furthermore, BDE forms the summer triangle.

37 Suppose a star has a peak emission wavelength of 290 nm, a radius of $1.711R_{\text{Sun}}$, an apparent magnitude

of −1.46, and an absolute magnitude of +1.43. What is the intensity (power per unit area) of a star as measured on Earth?

(A) $1.01 \times 10^{28}\,\mathrm{W\,m^{-2}}$

(B) $3.846 \times 10^{26}\,\mathrm{W\,m^{-2}}$

(C) $\mathbf{1.21 \times 10^{-7}\,W\,m^{-2}}$

(D) $1.15 \times 10^{26}\,\mathrm{W\,m^{-2}}$

(E) $3.89 \times 10^{15}\,\mathrm{W\,m^{-2}}$

Solution:

Answer: C

Using the Distance Modulus, we can find the distance to the star,

$$-1.46 - 1.43 = 5 \lg \frac{d}{10} \implies d = 10 \times 10^{-\frac{2.89}{5}} = 2.6424\,\mathrm{pc}.$$

Now we can find the luminosity of the star using the Stefan–Boltzmann law and Wien's displacement law:

$$L = 4\pi\left(1.711 \times 6.963 \times 10^{8}\right)^2 \left(5.67 \times 10^{8}\right)\left(\frac{2.898 \times 10^{-3}}{290 \times 10^{-9}}\right)^4 = 1.0085 \times 10^{28}\,\mathrm{W}.$$

We can then find the intensity at Earth using the inverse square law:

$$I = \frac{L}{4\pi d^2} = \frac{1.0085 \times 10^{28}}{4\pi(2.6424 \times 3.086 \times 10^{16})^2} = \boxed{1.21 \times 10^{-7}\,\mathrm{W\,m^2}}.$$

38 On 28th November 1967, from a newly commissioned radio telescope, Jocelyn Bell and her supervisor Antony Hewish detected a series of pulses, evenly spaced at every 1.337 seconds appearing at the same declination and right ascension for the following few weeks. They nicknamed the signal LGM-1, for "Little Green Men" (a playful name pointing to intelligent beings of extraterrestrial origin). Subsequently on 21st December 1967 they discovered a second signal originating from a different part of the sky. What was the origin of these signals?

(A) **Pulsars**

(B) Quasars

(C) A random terrestrial source

(D) Aliens sending a signal

(E) Black holes

Solution:

Answer: A

39 Figure 9 plots orbital inclination against perihelion distance for over 3800 comets. Circled are four clusters of comets with very similar orbital profiles. Why do these comets have almost identical orbits to each other?

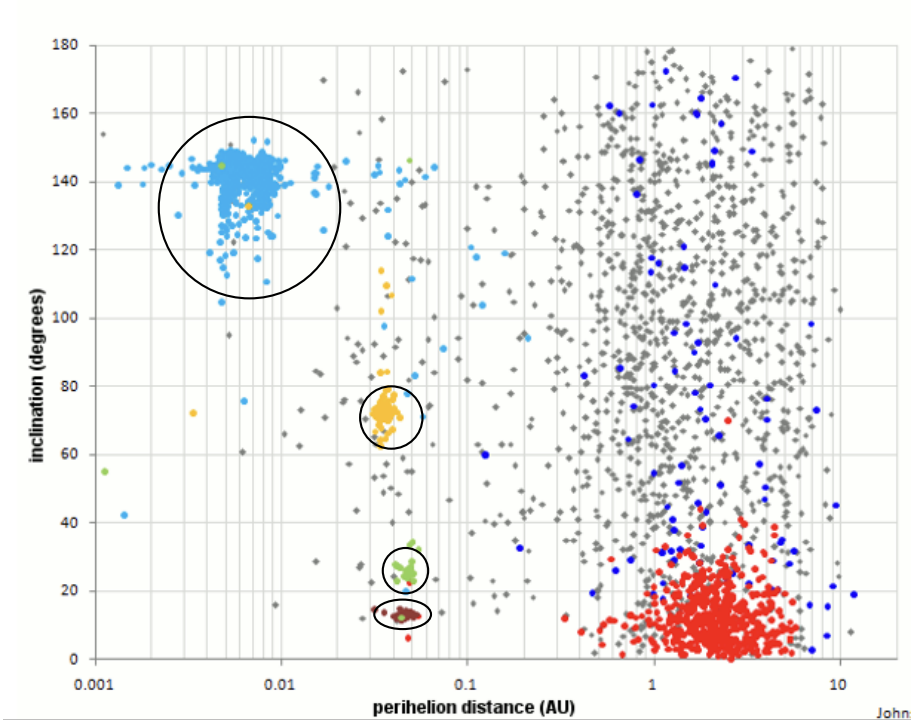

Figure 9: Question **39**.

(A) Since their perihelion distance is so small, they orbit the Sun tightly, thus causing these clusters.

(B) **Each cluster contains small comets we see today. They originated from a common parent comet that broke up into smaller pieces due to their proximity to the Sun at perihelion.**

(C) All the smaller comets became gravitationally bound to a larger comet within each of those clusters.

(D) The solar wind pushed these comets together into distinct clusters based on their masses.

(E) None of the above.

Solution:

Answer: B

40 The cosmic microwave background radiation (CMBR) is remnant electromagnetic radiation left by the Big Bang that has since been redshifted to microwave radiation equal to that emitted by a blackbody of

temperature 2.7 K. Which of the following options best describe how the CMBR came about?

(A) Matter and antimatter annihilated each other, emitting this light.

(B) Matter fell into primordial black holes, forming very hot accretion discs thus emitting light.

(C) **Until around 380,000 years after the Big Bang, the Universe was too hot to let light pass through as it would be scattered by rapidly moving protons and electrons. The CMBR is this light that was emitted when the Universe became just cool enough to let light pass through.**

(D) Actually, it does not exist. It was later found that pigeon droppings on the radio telescope used by Robert W. Wilson and Arno A. Penzias were the cause of this erroneous measurement.

(E) Light emitted from the fusion of protons.

Solution:

Answer: C

41 To observers on Earth, Saturn's rings appear to tilt forwards and backwards over many years; sometimes the rings appear edge-on and disappear entirely. Which of the following best explains this phenomenon?

(A) Saturn undergoes axial precession over a period of 32 years, which causes the rotation axis of the planet to tilt towards and away from Earth.

(B) Saturn's rings are actually tilted relative to its equator. This misalignment between the ring and its equatorial bulge causes the rings to precess relative to the planet.

(C) The orbit of Saturn is inclined relative to Earth, therefore depending on the relative position of Saturn and Earth in their orbits, an observer on Earth might be viewing Saturn from the North or from the South.

(D) **Saturn's rotational axis appears to tilt towards and away from the Sun as it moves around in its orbit, similar to how axial tilt causes seasons on Earth.**

(E) No, this phenomenon does not actually happen. The source of this myth was an erroneous report by amateur astronomers mistaking Jupiter for Saturn.

Solution:

(A) is wrong because axial precession is too slow.

(B) is wrong because Saturn's rings are aligned with the equator of the planet.

(C) is wrong, because the relative inclination between the Earth and Saturn is only 2.5 degrees, too small to account for this phenomenon.

(D) is correct.

(E) is just wrong.

42 Open clusters tend to disperse very quickly within a few million years after their formation, while globular

clusters tend to remain bound to each other for very long periods of time. Which of the following best explains this difference?

(A) **Open clusters have a lower overall mass and density compared to globular clusters, therefore an object escaping from an open cluster will require a lower escape velocity than if it were escaping from a globular cluster.**

(B) Open clusters lie in the rotating spiral arms of the galaxy, where the differential rotation of the arms causes open clusters to shear apart quickly relative to globular clusters which reside further away in the galactic halo.

(C) Open clusters tend to have a larger population of O- and B-type stars, which have very strong stellar winds. This generates significant radiation pressure which tends to push away members of the open cluster over millions of years.

(D) Open clusters are closer to the galactic centre compared to globular clusters, thus they experience greater tidal forces due to the galactic bulge, which tends to tear apart the cluster.

(E) No, the statement is false. Open clusters are just as long-lived as globular clusters, and do not disperse quickly.

Solution:

(A) is correct.

(B) is false because open clusters are far smaller than the length scale at which differential rotation of the spiral arms and galactic tidal forces act.

(C) is false, because although the stellar wind is sufficiently strong to expel remnant gas in the cluster, it is not strong enough to significantly affect the orbit of stars.

(D) is false for the same reason as B.

(E) is false.

43 In colour-magnitude diagrams of star clusters, a feature called the main sequence turnoff point can often be identified. This is a "kink" in the main sequence that suddenly turns into the red giant branch. The position of the main sequence turnoff point can be used to estimate the age of the star cluster.

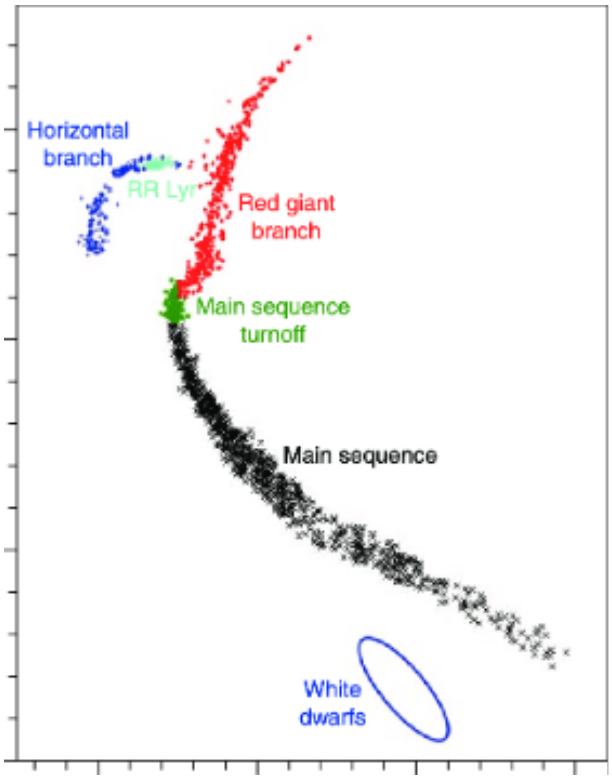

Figure 10: Question **43**.

Which of the following best explains why the position of the main sequence turnoff point is related to the age of the star cluster?

(A) In the early Universe, star clusters are formed with less metals in their stellar atmospheres. Star clusters with more metals tend to form a lower percentage of high-mass stars, resulting in the main sequence turnoff point.

(B) The main sequence turnoff point reflects the point at which a star accretes enough mass to directly fuse elements heavier than hydrogen. Since accretion grows with time, the main sequence turnoff point reflects the age of the cluster.

(C) High-mass stars undergo a supernova explosion at the main sequence turnoff point.

(D) **The main sequence lifetime of stars decreases with mass. Thus, as a star cluster ages, its stars start evolving off the main sequence beginning with the most massive stars. Therefore, in older clusters, the main sequence turnoff is located further down the main sequence track.**

(E) The main sequence lifetime of stars increases with mass. Thus, as a star cluster ages, its stars start evolving off the main sequence beginning with the least massive stars. Therefore, in older clusters, the main sequence turnoff is located further down the main sequence track.

Solution:

Answer: D.

A, B, C are nonsense answers that do not explain any clear link to the age of the cluster. E is wrong; for more massive stars, the rate of fusion in the core is much higher, resulting in a shorter main

sequence lifetime.

44 One clear night in Singapore, your astronomy club is out stargazing to catch Jupiter and Saturn at opposition. Suddenly, a club junior turns to you and asks, "can we see Mars anytime soon?" You take out your trusty old star chart mobile application, and found that Mars will not be visible in the skies for more than a year.

"But we can see Jupiter and Saturn nearly every year!" your junior protests.

How should you explain this phenomenon?

(A) Mars has an orbital period of 1.8 years, therefore it will take 1.8 years for Mars to reappear between oppositions.

(B) Mars is in a 2:1 orbital resonance with Jupiter. Therefore, whenever Jupiter is at opposition, Mars will be at conjunction, and thus cannot be seen.

(C) This is due to Earth's axial tilt. In certain years, at certain locations on Earth, Mars does not rise above the horizon when it is at opposition because it would be at the southernmost point of the ecliptic.

(D) Mars has a synodic period of 2.1 years with Earth, while Jupiter and Saturn have much shorter synodic periods with Earth, closer to 1 year. Therefore, Jupiter and Saturn can be seen nearly on a yearly basis, while Mars is only visible every other year.

(E) Mars is no longer visible, because Elon Musk has blown up the planet in his efforts to colonise it.

Solution:

Answer: D

B and E are clearly nonsense answers. C is a possible scenario at very high latitudes, but not in Singapore. B is not correct, because the time between oppositions is determined by the synodic period rather than the orbital period.

45 In a GP essay about the implications of finding alien life, John ends his conclusion with "It is possible that far away, on a planet orbiting the star A, there exists an alien civilisation. Perhaps those aliens observe our Sun dimming on a regular basis the same way we observe their Sun dimming." Which star is the most **UNLIKELY** to be star A?

(A) Spica

(B) Aldebaraan

(C) Castor

(D) Polaris

(E) Regulus

Solution:

Answer: D

This question tests knowledge on the ecliptic. The word 'regular' implies that the dimming occurs from the planet obscuring the observer's view of the star. For this to happen, the observer must be on the same plane as the star and the planet. For us, that plane is the ecliptic. Hence, the only star that does not fit this statement is a star that is not in a zodiac constellation because the zodiac lie on the ecliptic.

46 In the spectrum of distant quasars, it is typical to find a peculiar absorption called a *Lyman-Alpha Forest*, which consists of a region of densely-packed absorption lines, creating a distinct "treeline" profile. The Lyman-Alpha forest terminates at a strict cutoff on the right (corresponding to longer wavelengths).

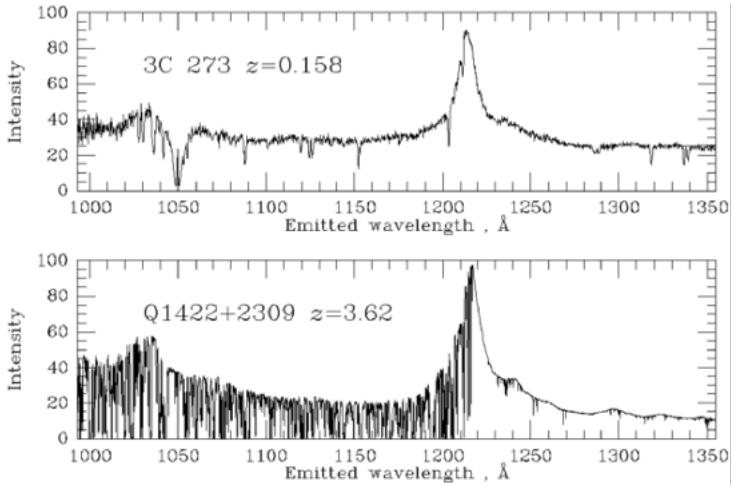

Figure 11: Question **46**.

What information can we determine from this cutoff point?

(A) The mass of the quasar.

(B) **The age of the quasar.**

(C) The brightness of the quasar.

(D) The metallicity of the quasar.

(E) The chemical composition of hydrogen clouds in the Universe.

Solution:

B. The cutoff point reflects the original H-α absorption line near the origin of the quasar, which has been redshifted as the light propagates through the expanding universe; the redshift can be calculated by comparing with the non-redshifted wavelength of the hydrogen absorption line, which in turn tells us the age or distance of the quasar (given a background cosmological model). As the universe expands, the overall spectrum is redshifted and moved to the right, while absorption by hydrogen clouds create new absorption features on the left. Therefore, this cutoff point is a

right-cutoff.

47 Which of the following statements about moons in the Solar System is **FALSE**?

(A) Tidal phenomenon on Earth is mainly caused by the Moon.

(B) Three of the Galilean moons, Io, Europa and Ganymede, are in orbital resonance with each other.

(C) Mercury and Venus do not have moons because they are too close to the Sun.

(D) Saturn's shepherd moons are responsible for maintaining the shape of the rings of Saturn.

(E) **The Moon is gradually getting closer to Earth due to tidal forces.**

Solution:

Answer: E

48 Which of the following statements is always true?

(A) Assuming ideal viewing conditions (where the atmosphere is absent), the portion of the night sky visible to an observer closer to the poles is always a subset of the portion seen by an observer on the equator.

(B) The day of the latest sunrise, using civil time, occurs on the date of the hemisphere's winter solstice.

(C) In the northern hemisphere, turning to face the direction of the Sun when reaches its maximum altitude (at solar noon) makes you face the south.

(D) **Using an equatorial mount that is properly setup to track the stars will cause the Moon to have an apparent eastward drift when centered on it.**

(E) Three observers, all at the same latitude but have their longitude separated by 120° will never see the Sun simultaneously as at least one will be experiencing night time.

Solution:

(A) has one edge case: The celestial poles are not visible to an equatorial observer. Ironically by including the atmosphere, the statement becomes true due to atmospheric refraction.

(B) is not true. The shortest day occurs on the Winter Solstice but it does not mention anything about the latest sunrise. This can occur a few days before or after the solstice due to Earth;s obliquity, orbital eccentricity etc. To predict the day of latest sunrise, one needs to incorporate the Equation of Time.

(C) is not always true! Consider the case on the day of the Summer Solstice. Any observer with latitude $\phi < 23.5^\circ$ will be facing North instead.

(D) is true. The Moon's synodical period with a stationary observer is longer than 24 hours. It will thus seem to" lag" behind the stars so with a tracking mount will slowly start drifting eastwards.

(E) is not always true! Consider the case where the three observers are experiencing the Midnight Sun. The is circumpolar for all three observers and will see the Sun simultaneously.

49 Which of the following statement(s) is/are true?

I The maximum change in velocity that is ever required in an inclination change manoeuvre is during a change from an equatorial orbit to a polar orbit or vice versa.

II Assuming that all the planets are in circular orbits around the Sun, if we magically and instantly halve the Sun's mass, the planets will thereafter travel on parabolic trajectories.

III Comparing between closed orbits with the same orbital period, the circular orbit allows an orbiting satellite to have the maximum angular momentum.

IV If we treat the whole Solar System as an isolated system, the separation between the Sun's centre and the barycentre of the Solar System will stay constant because the barycentre does not accelerate.

(A) **I** and **II**

(B) **I** and **III**

(C) **I** and **IV**

(D) **II and III**

(E) **II** and **IV**

Solution:

I is false. The maximum Δv needed is when you do transition from a prograde to a retrograde orbit.

II is true. The initial velocity of orbit is given by

$$v_0 = \sqrt{\frac{GM_{\text{Sun}}}{r}}.$$

Because the mass of the Sun suddenly halves, the new escape velocity becomes

$$v_{\text{esc}} = \sqrt{\frac{2G(0.5M_{\text{Sun}})}{r}} = v_0.$$

Thus, all planets will now be on an escape trajectory.

III is true. One can compare the angular momenta and energy to make the conclusion. Alternatively, one can use the result

$$L = \mu\sqrt{GMa(1-e^2)}$$

in which for the same semi-major axis (and orbital period), the angular momentum is maximised when $e = 0$.

IV is false. The barycentre of the planets and the Sun is the centre of mass of the system. Unless acted upon by an external force, this point does not accelerate.

50 A sailor out in the open ocean takes the following measurements. Which of the following measurement is **NOT** required for the sailor to accurately determine their longitude?

(A) Civil time of local sunset

(B) Civil time of sunset in Greenwich

(C) **Declination of the sun**

(D) Timezone used for civil time

(E) Height above sea level

Solution:

We need to correct for height above sea level as that delays sunset the higher above sea level you go. So that is required. After correcting for it, we get the time in Greenwich when the sun sets from the time of local sunset and timezone used

$$T_{\text{GMT}} = T_{\text{SS,Local}} - \delta T_{TZ}.$$

If you're in Singapore Civil Time or (GMT+8), $\delta T_{TZ} = 8$. We can then find the time difference between the sailor's sunset and the time of sunset at Greenwich to be

$$\Delta T = T_{\text{SS,GMT}} - T_{\text{GMT}}.$$

A positive ΔT means that the Sun sets at the sailor's position before Greenwich. That would give us an eastern longitude. Then we find the longitude with

$$\theta = \frac{360^\circ}{24\,\text{h}} \times \Delta T.$$

Again, if $\theta > 0$, the sailor is East of Greenwich. Note that the declination of the Sun is not used here

as we already have given the time of sunset at Greenwich.

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